chapter 11 Worksheet

Key

Disclaimer: The actual exam differs.

Solve.

1) $3x^2 = 45$

$x^2 = \frac{45}{3}$

$x^2 = 15$

$x = \pm\sqrt{15}$

2) $49x^2 + 25 = 0$

$49x^2 = -25$

$x^2 = \frac{-25}{49}$

$x = \pm\sqrt{-25/49}$

$x = \pm\frac{5}{7}i$

3) $\left(x + \frac{2}{5}\right)^2 = \frac{23}{25}$

$x + 2/5 = \pm\sqrt{\frac{23}{25}}$

$x + \frac{2}{5} = \pm\frac{\sqrt{23}}{5}$

$x = -\frac{2}{5} \pm \frac{\sqrt{23}}{5}$

4) Let $f(x) = (x + 2)^2$. Find x so that $f(x) = 42$.

$(x+2)^2 = 42$

$x+2 = \pm\sqrt{42}$

$x = -2 \pm\sqrt{42}$

Solve by completing the square.

5) $z^2 + 14z + 38 = 0$

$z^2 + 14z = -38$

$z^2 + 14z + 49 = -38 + 49$

$(z+7)^2 = 11$

$(z+7) = \pm\sqrt{11}$

$z = -7 \pm \sqrt{11}$

6) $2x^2 + 7x + 3 = 0$

$2x^2 + 7x = -3$

$x^2 + \frac{7}{2}x = \frac{-3}{2}$

$x^2 + \frac{7}{2}x + \frac{49}{16} = \frac{-3}{2} + \frac{49}{16}$

$(x + 7/4)^2 = -\frac{24}{16} + \frac{49}{16}$

$(x + 7/4)^2 = 25/16$

$x + \frac{7}{4} = \pm\sqrt{25/16}$

$x + \frac{7}{4} = \pm 5/4$

$x = -\frac{7}{4} \pm \frac{5}{4}$

$-\frac{7}{4} + \frac{5}{4} = -\frac{2}{4} = \boxed{-\frac{1}{2}}$

$-\frac{7}{4} - \frac{5}{4} = -\frac{12}{4} = \boxed{-3}$

7) $x^2 + 8x = 3$

$x^2 + 8x + 16 = 3 + 16$

$(x+4)^2 = 19$

$x + 4 = \pm\sqrt{19}$

$x = -4 \pm \sqrt{19}$

Complete the square to find the x-intercepts of the function.

8) $f(x) = 5x^2 - 4x - 9$

$5x^2 - 4x - 9 = 0$

$5x^2 - 4x = 9$

$x^2 - \frac{4}{5}x = \frac{9}{5}$

$x^2 - \frac{4}{5}x + \frac{4}{25} = \frac{9}{5} + \frac{4}{25}$

$(x - 2/5)^2 = \frac{45}{25} + \frac{4}{25}$

$(x - 2/5)^2 = \frac{49}{25}$

$x - 2/5 = \pm\sqrt{49/25}$

$x - 2/5 = \pm 7/5$

$x = \frac{2}{5} \pm \frac{7}{5}$

$\frac{2}{5} + \frac{7}{5} = \frac{9}{5}$

$\frac{2}{5} - \frac{7}{5} = \frac{-5}{5} = -1$

x-int

$(9/5, 0)$ and $(-1, 0)$

Use $A = P(1 + r)^t$ to find the interest rate. Round to the nearest hundredth, if necessary.

9) \$5000 grows to \$6161 in 2 years. Assume that interest is compounded annually.

A) 0.11% B) 1.11% C) 1.23% D) 11%

$6161 = 5000(1+r)^2$

$\frac{6161}{5000} = (1+r)^2$

$\pm\sqrt{\frac{6161}{5000}} = 1+r$

$-1 \pm \sqrt{\frac{6161}{500}} = r$

rates must be positive

so $r \approx -1 + \sqrt{6161/5000}$

$r \approx .110$

$= 11\%$

Solve.

10) $2m^2 + 8m + 3 = 0$

$a=2 \quad b=8 \quad c=3$

$m = \frac{-8 \pm \sqrt{64-4(2)(3)}}{2(2)}$

$= \frac{-8 \pm \sqrt{64-24}}{4}$

$= \frac{8 \pm \sqrt{40}}{4}$

$= \frac{8 \pm 2\sqrt{10}}{4}$

$= 2 \pm \frac{1}{2}\sqrt{10}$

11) $\frac{6}{x} + \frac{6}{x+9} = 1$

$x(x+9)\left(\frac{6}{x} + \frac{6}{x+9}\right) = 1(x)(x+9)$

$6(x+9) + 6x = x^2 + 9x$

$6x + 54 + 6x = x^2 + 9x$

$12x + 54 = x^2 + 9x$

$0 = x^2 - 3x - 54$

$(x-9)(x+6) = 0$

$x-9=0 \quad x+6=0$

$x=9 \quad x=-6$

12) $14(x - 4) - (x - 22) = (x + 2)(x - 4)$

$14x - 56 - x + 22 = x^2 - 2x - 8$

$13x - 34 = x^2 - 2x - 8$

$0 = x^2 - 15x + 26$

$0 = (x-13)(x-2)$

$x-13=0 \quad x-2=0$

$x=13 \quad x=2$

13) $x^3 - 27 = 0$

$(x-3)(x^2+3x+9)=0$

$x-3=0$ $\qquad$ $x^2+3x+9=0$

$x=3$ $\qquad$ $x = \frac{-3 \pm \sqrt{9-4(1)(9)}}{2}$ $\qquad$ $x = \frac{-3 \pm 3i\sqrt{3}}{2}$

$x = \frac{-3 \pm \sqrt{9-36}}{2}$

SOLUTIONS

$x = \frac{-3 \pm \sqrt{-27}}{2}$ $\qquad$ $\{3, \frac{-3 \pm 3i\sqrt{3}}{2}\}$

Use the discriminant to determine whether the following equations have solutions that are: two different rational solutions; two different irrational solutions; exactly one rational solution; or two different imaginary solutions.

14) $s^2 + 4s - 5 = 0$

$a=1$ $\quad$ $b=4$ $\quad$ $c=-5$

2 DIFFerent rational SOLUTIONS

$D = b^2 - 4ac$

$D = 16 - 4(1)(-5)$

$D = 16 + 20$

$D = 36$ $\quad$ A perfect square

15) $t^2 + 4t + 4 = 0$

$a=1$ $\quad$ $b=4$ $\quad$ $c=4$

one rational SOLUTION

$D = b^2 - 4ac$

$D = 16 - 4(1)(4)$

$D = 16 - 16$

$D = 0$

16) $w^2 - 3w + 8 = 0$

$a=1$ $\quad$ $b=-3$ $\quad$ $c=8$

2 different imaginary SOLUTIONS.

$D = b^2 - 4ac$

$D = 9 - 4(1)(8)$

$D = 9 - 32$

$D = = -23 < 0$

Write a quadratic equation having the given numbers as solutions. (using integer coefficients

17) $-\frac{1}{4}, -\frac{2}{3}$

$x = -1/4$ $\quad$ $x = -2/3$

$4x = -1$ $\quad$ $3x = -2$

$4x + 1 = 0$ $\quad$ $3x + 2 = 0$

$(4x+1)(3x+2) = 0$

$12x^2 + 8x + 3x + 2 = 0$

$12x^2 + 11x + 2 = 0$

18) 8, only solution

$x = 8$

$x - 8 = 0$

$(x-8)^2 = 0$

$x^2 - 16x + 64 = 0$

19) $4 - \sqrt{3}, 4 + \sqrt{3}$

$x = 4 - \sqrt{3}$ $\quad$ $x = 4 + \sqrt{3}$

$x - 4 + \sqrt{3} = 0$ $\quad$ $x - 4 - \sqrt{3} = 0$

$((x-4)+\sqrt{3})((x-4)-\sqrt{3}) = 0$

$(x-4)^2 - 3 = 0$

$x^2 - 8x + 16 - 3 = 0$

$x^2 - 8x + 13 = 0$

Solve the formula for the indicated letter. Assume that all variables represent nonnegative numbers.

20) $A = \frac{1}{3}\pi r^2$ for r

$3A = \pi r^2$

$\frac{3A}{\pi} = r^2$

$\sqrt{\frac{3A}{\pi}} = r$ $\quad$ since $r \geq 0$

21) $rm = t^2 - mt$, for t

$0 = t^2 - mt - rm$

$a = 1 \quad b = -m \quad c = -rm$

$t = \dfrac{m \pm \sqrt{m^2 - 4(1)(-rm)}}{2(1)}$

$t = \dfrac{m \pm \sqrt{m^2 + 4rm}}{2}$

Solve.

22) $(t + 2)^{2/3} + 3(t + 2)^{1/3} - 10 = 0$

Let $u = (t+2)^{1/3} \rightarrow u^2 = (t+2)^{2/3}$

$u^2 + 3u - 10 = 0$

$(u+5)(u-2) = 0$

$u + 5 = 0 \quad u - 2 = 0$

$u = -5 \quad u = 2$

$(t+2)^{1/3} = -5$

$t + 2 = -125$

$\boxed{t = -127}$

$(t+2)^{1/3} = 2$

$t + 2 = 8$

$\boxed{t = 6}$

$\{-127, 6\}$

23) $x^4 - 12x^2 + 27 = 0$

Let $u = x^2 \rightarrow u^2 = x^4$

$u^2 - 12u + 27 = 0$

$(u - 9)(u - 3) = 0$

$u - 9 = 0 \quad u - 3 = 0$

$u = 9 \quad u = 3$

$x^2 = 9 \quad x^2 = 3$

$x = \pm 3 \quad x = \pm\sqrt{3}$

$\{3, -3, \sqrt{3}, -\sqrt{3}\}$

24) $\left(x^2 - 7\right)^2 + 7\left(x^2 - 7\right) + 10 = 0$

Let $u = x^2 - 7$

$u^2 + 7u + 10 = 0$

$(u+5)(u+2) = 0$

$u + 5 = 0 \quad u + 2 = 0$

$u = -5 \quad u = -2$

$x^2 - 7 = -5 \quad x^2 - 7 = -2$

$x^2 = 2 \quad x^2 = 5$

$x = \pm\sqrt{2} \quad x = \pm\sqrt{5}$

$\{\pm\sqrt{2}, \pm\sqrt{5}\}$

Without graphing, find the vertex.

25) $f(x) = -(x + 3)^2 + 3$

$y = a(x-h)^2 + k$

(h, k) is vertex

vertex is $(-3, 3)$

Find the axis of symmetry of the graph of the parabola.

26) $f(x) = -(x + 4)^2 + 8$

$x = -4$ is the axis of symmetry

Without graphing, find the maximum value or minimum value.

27) $f(x) = 0.67(x + 6)^2 - 7$

vertex $= (-6, -7)$ faces up

So min value is -7

Complete the square to write the function in the form $f(x) = a(x - h)^2 + k$.

28) $f(x) = -x^2 - 4x - 3$

$f(x) = -(x^2 + 4x \quad) - 3$

$f(x) = -(x^2 + 4x + 4) - 3 + 4$

$f(x) = -(x+2)^2 + 1$

29) $f(x) = 11x^2 + 4x + 5$

$f(x) = 11\left(x^2 + \frac{4}{11}x \quad \right) + 5$

$f(x) = 11\left(x^2 + \frac{4}{11}x + \frac{4}{121}\right) + 5 - \frac{4}{11}$

$f(x) = 11\left(x + \frac{2}{11}\right)^2 + \frac{55}{11} - \frac{4}{11}$

$f(x) = 11\left(x + \frac{2}{11}\right)^2 + \frac{51}{11}$

Find the vertex.

30) $f(x) = 2x^2 - 4x - 2$

$x = \frac{-b}{2a} = \frac{4}{2(2)} = \frac{4}{4} = 1$ $\qquad$ $V\ (1, -4)$

$f(1) = 2(1)^2 - 4(1) - 2$

$= 2 - 4 - 2$

$= 2 - 6$

$= -4$

Find the line of symmetry.

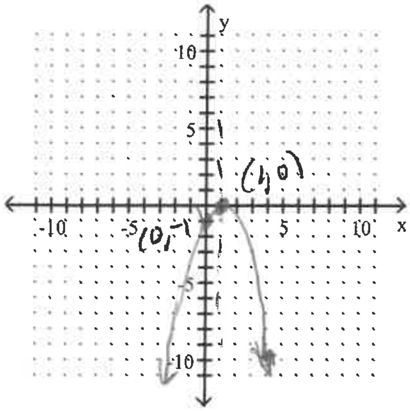
31) $f(x) = 3x^2 - 18x + 22$

$x = -\frac{b}{2a} = \frac{-(-18)}{2(3)} = \frac{18}{6} = 3$

$x = 3$

Graph. Label the vertex, axis of symmetry and at least one other point on the graph.

32) $f(x) = -x^2 + 2x - 1$



$y = -(x^2-2x+1)+1-1$

$y = -(x-1)^2$

$V(1,0)$

faces Down

$(0,-1)$

Find the x- and y-intercepts. If no x-intercepts exist, state so.

33) $f(x) = 6x^2 + 10x + 2$

$f(0)=2$ $(0,2)$ y-intercept

$6x^2+10x+2=0$

$X = \dfrac{-10 \pm \sqrt{100-4(6)(2)}}{2(6)}$

$X = \dfrac{-10 \pm \sqrt{100-48}}{12}$

$X = \dfrac{-10 \pm \sqrt{52}}{12}$

$X = \dfrac{-10 \pm 2\sqrt{13}}{12}$

$X = \dfrac{-5 \pm \sqrt{13}}{6}$

$\left(\dfrac{-5+\sqrt{13}}{6}, 0\right)$ $\left(\dfrac{-5-\sqrt{13}}{6}, 0\right)$ X-INT

Solve.

34) The length and width of a rectangle have a sum of 72. What dimensions give the maximum area?

$L+W=72$

$W=72-L$

$Area = LW$

$Area = L(72-L)$

$Area = 72L-L^2$

has a max value at the vertex

$Area = -L^2+72L$

$L = \dfrac{-72}{-2} = 36$

$W = 72-L = 36$

35) What is the minimum product of two numbers whose difference is 76?

$x - y = 76$

$x = 76 + y$

product $= xy$

product $= x(y) = (76+y)(y) = 76y + y^2$

min at vertex $\quad y = \frac{-b}{2a} = \frac{-76}{2} = -38$

$x = 76 - 38 = 38$

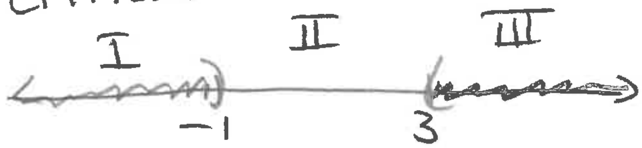
product $= (38)(-38) = -1444$

Solve. Provide answers in interval notation for all of the following inequalities

36) $p^2 - 4p + 3 > 0$

$(p-3)(p+1) > 0$

critical #s $\quad p = 3 \quad p = -1$



I test $p = -2$

$(-2-3)(-2+1) > 0$ true

II test $p = 0$

$(0-3)(0+1) > 0$

false

III test $p = 4$

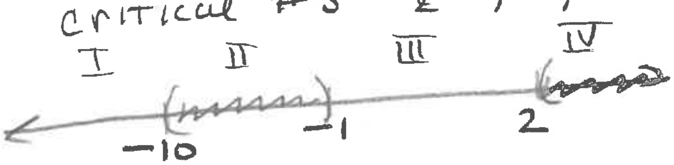
$(4-3)(4+1) > 0$

true

$\boxed{(-\infty, -1) \cup (3, \infty)}$

37) $(a + 10)(a + 1)(a - 2) > 0$

critical #s $\{-10, -1, 2\}$



I-test $a = -11$

$(-11+10)(-11+1)(-11-2) > 0$

false

II test $a = -2$

$(-2+10)(-2+1)(-2-2) > 0$ true

III test $a = 0$

$(0+10)(0+1)(0-2) > 0$ false

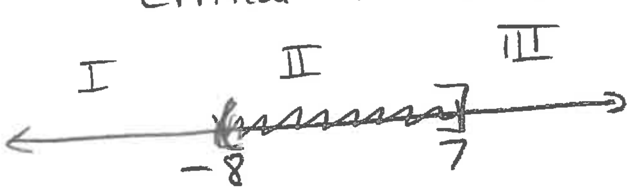
IV test $a = 3$

$(3+10)(3+1)(3-2) > 0$ true

Solution $\boxed{(-10, -1) \cup (2, \infty)}$

38) $\frac{x - 7}{x + 8} \le 0$

critical numbers $7, -8$



I test $x = -9$

$\frac{(-9-7)}{-9+8} \le 0$ false

II test $x = 0$

$\frac{0-7}{0+8} \le 0$ true

III test $x = 8$

$\frac{8-7}{8+8} \le 0$ false

Solution $[-8, 7]$

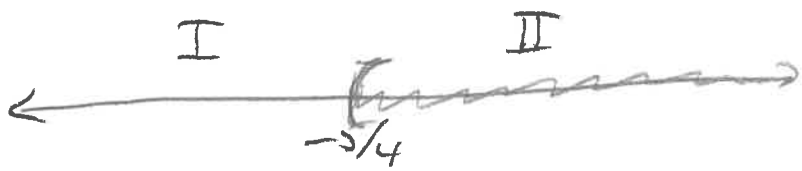
39) $\dfrac{4x+5}{2x^2+5} > 0$

Since $2x^2+5 \neq 0$

only critical # is where

$4x+5=0$

$x = -5/4$



I TEST $x = -2$

$$\frac{(4)(-2)+5}{(2(-2)^2+5)} = \frac{-8+5}{4+5} > 0 \quad \text{FALSE}$$

II TEST $x = 0$

$$\frac{4(0)+5}{2(0)^2+5} > 0 \quad \text{TRUE}$$

SOLUTION $(-5/4, \infty)$