

Do all work neatly and systematically for full credit. Total points: 104 (#1 - #12, 3 points each; #13 - #30, 4 points each)

Factor the polynomial completely. If the polynomial cannot be factored, say it is prime.

1) $28x^5 + 8x^3 - 20x$

$$= 4x(7x^4 + 2x^2 - 5)$$

$$= 4x(7x^2 - 5)(x^2 + 1)$$

2) $xy + 8x - 5y - 40$

$$= x(y + 8) - 5(y + 8)$$

$$= (y + 8)(x - 5)$$

3) $x^2 + 11x + 18$

$$= (x + 9)(x + 2)$$

4) $9x^2 + 11x - 14$

$$= (9x - 7)(x + 2)$$

5) $9x^2 - 16$

$$= (3x)^2 - 4^2$$

$$= (3x + 4)(3x - 4)$$

6) $15(x + 7) - y(x + 7)$

$$= (x + 7)(15 - y)$$

7) $3x^2y + 12xy - 63y$

$$= 3y(x^2 + 4x - 21)$$

$$= 3y(x + 7)(x - 3)$$

8) $x^2 - x - 48$

prime

9) $3x^3 - 3$

$$= 3(x^3 - 1)$$

$$= 3(x - 1)(x^2 + x + 1)$$

10) $72x^3 - 632x^2 + 448x$

$$= 8x(9x^2 - 79x + 56)$$

$$= 8x(x - 8)(9x - 7)$$

$$\begin{array}{r} 1 \overline{) 872} \\ \underline{8} \\ 7 \\ \underline{7} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

11) $10x^2 + 21x + 9$

$$= (2x + 3)(5x + 3)$$

$$\begin{array}{r} 2 \quad 3 \quad 15 \\ 5 \times 3 \quad \frac{6}{21} \end{array}$$

12) $25x^2 - 60xy + 36y^2$

$$= (5x - 6y)(5x - 6y)$$

$$= (5x - 6y)^2$$

List all numbers for which the rational expression undefined.

$$13) \frac{x^2 - 4}{15x^2 + 8x + 1}$$

$$= \frac{(x+2)(x-2)}{(3x+1)(5x+1)}$$

$$3x+1=0 \quad \text{or} \quad 5x+1=0$$

$$x = -\frac{1}{3} \quad x = -\frac{1}{5}$$

Solve the equation by factoring.

$$14) 3x^2 = 4x + 7$$

$$3x^2 - 4x - 7 = 0$$

$$(3x-7)(x+1) = 0$$

$$3x-7=0 \quad \text{or} \quad x+1=0$$

$$x = \frac{7}{3} \quad x = -1$$

$$\left\{ -1, \frac{7}{3} \right\}$$

Simplify the rational expression.

$$15) \frac{2x+2}{10x^2+16x+6}$$

$$= \frac{2(x+1)}{2(5x^2+8x+3)}$$

$$= \frac{\cancel{2}(x+1)}{\cancel{2}(5x+3)(x+1)}$$

$$= \frac{1}{5x+3}$$

Perform the indicated operation and simplify if possible.

$$16) \frac{5}{x+1} \cdot (3x+3)$$

$$= \frac{5}{\cancel{x+1}} \cdot \frac{3(\cancel{x+1})}{1}$$

$$= 15$$

Simplify the rational expression.

$$17) \frac{b-a}{a^2-b^2}$$

$$= \frac{-(a-b)}{(a+b)(\cancel{a-b})}$$

$$= -\frac{1}{a+b}$$

Simplify, if possible.

$$18) \frac{t^2-4t+4}{t^2+4t+4} \div \frac{(t-2)}{(t+2)} + \frac{3t+6}{t^2-4}$$

$$= \frac{(t-2)(\cancel{t-2})}{(\cancel{t+2})(t+2)} \cdot \frac{\cancel{t+2}}{\cancel{t-2}} \cdot \frac{(t+2)(t-2)}{3(\cancel{t+2})}$$

$$= \frac{(t-2)^2}{3(t+2)}$$

Simplify.

$$19) \frac{x+4+\frac{4}{x}}{x+7+\frac{10}{x}} \quad \text{LCD: } x$$

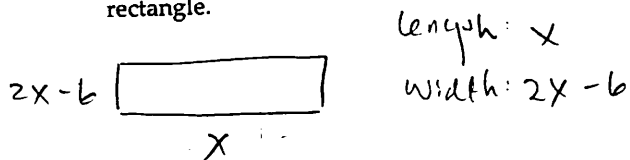
$$= \frac{x(x+4+\frac{4}{x})}{x(x+7+\frac{10}{x})}$$

$$= \frac{x^2+4x+4}{x^2+7x+10}$$

$$2) = \frac{(x+2)(\cancel{x+2})}{(x+5)(\cancel{x+2})} = \frac{x+2}{x+5}$$

Solve the problem.

- 20) The width of a rectangle is 6 kilometers less than twice its length. If its area is 140 square kilometers, find the dimensions of the rectangle.



$$x(2x - 6) = 140$$

$$2x^2 - 6x - 140 = 0$$

$$2(x^2 - 3x - 70) = 0$$

$$2(x - 10)(x + 7) = 0$$

$$x - 10 = 0 \text{ or } x + 7 = 0$$

$$x = 10 \quad x = -7 \text{ (Reject)}$$

So, the dimensions are 10 km by 7 km.

Simplify the rational expression.

$$21) \frac{4m^3 - 4m^2 - 8m}{m^2 - 3m + 2}$$

$$= \frac{4m(m^2 - m - 2)}{(m - 2)(m - 1)}$$

$$= \frac{4m(m - 2)(m + 1)}{(m - 2)(m - 1)} = \frac{4m(m + 1)}{m - 1}$$

Perform the indicated operation and simplify if possible.

$$22) \frac{y^2 - 10y + 24}{2y + 12} \cdot \frac{y + 6}{3y - 12}$$

$$= \frac{(y - 4)(y - 6)}{2(y + 6)} \cdot \frac{y + 6}{3(y - 4)}$$

$$= \frac{y - 6}{6}$$

Perform the indicated operation. Simplify, if possible.

$$23) \frac{3x + 24}{x^2 - 2x - 8} - \frac{x + 20}{x^2 - 2x - 8}$$

$$= \frac{3x + 24 - x - 20}{x^2 - 2x - 8}$$

$$= \frac{2x + 4}{(x - 4)(x + 2)}$$

$$= \frac{2(x + 2)}{(x - 4)(x + 2)} = \frac{2}{x - 4}$$

Perform the indicated operation and simplify if possible.

$$24) \frac{x^2 - 25}{x^2 - 5x} \div \frac{x^2 + 4x + 1}{2x + 8}$$

$$= \frac{(x - 5)(x + 5)}{x(x - 5)} \cdot \frac{2(x + 4)}{(x^2 + 4x + 1)}$$

$$= \frac{2(x + 4)(x + 5)}{x(x^2 + 4x + 1)}$$

Solve the equation by factoring.

$$25) 25t^3 - 16t = 0$$

$$t(25t^2 - 16) = 0$$

$$t(5t + 4)(5t - 4) = 0$$

$$t = 0 \text{ or } 5t + 4 = 0 \text{ or } 5t - 4 = 0$$

$$5t = -4$$

$$5t = 4$$

$$t = -\frac{4}{5}$$

$$t = \frac{4}{5}$$

$$\left\{ 0, -\frac{4}{5}, \frac{4}{5} \right\}$$

Perform the indicated operation and simplify if possible.

$$\begin{aligned}
 26) \frac{7a}{a^2+2a-8} - \frac{2}{a+4}, (x+1)(x-1) \\
 = \frac{7a}{(a+4)(a-2)} - \frac{2}{(a+4)} \cdot \frac{(a-2)}{(a-2)} \\
 = \frac{7a}{(a+4)(a-2)} - \frac{2a-4}{(a+4)(a-2)} \\
 = \frac{5a+4}{(a+4)(a-2)}
 \end{aligned}$$

$$27) \frac{16}{x^2-1} + \frac{8}{x+1} \quad \text{LCD} \cdot (x+1)(x-1)$$

$$\begin{aligned}
 = \frac{16}{(x+1)(x-1)} + \frac{8}{x+1} \cdot \frac{(x-1)}{(x-1)} \\
 = \frac{16}{(x+1)(x-1)} + \frac{8x-8}{(x+1)(x-1)} \\
 = \frac{8x+8}{(x+1)(x-1)} \\
 = \frac{8(x+1)}{(x+1)(x-1)} = \frac{8}{x-1}
 \end{aligned}$$

Solve the equation.

$$28) \frac{5}{y-4} = \frac{2}{y+4}$$

$$\text{LCD} : (y-4)(y+4), y \neq 4, y \neq -4$$

$$(y-4)(y+4) \cdot \frac{5}{y-4} = (y-4)(y+4) \cdot \frac{2}{y+4}$$

$$5(y+4) = 2(y-4)$$

$$5y+20 = 2y-8$$

$$5y-2y = -8-20$$

$$3y = -28 \quad \left\{ \begin{array}{l} -28 \\ 3 \end{array} \right\}$$

$$y = -\frac{28}{3}$$

Perform the indicated operation and simplify if possible.

$$\begin{aligned}
 29) \frac{x+5}{x^2+11x+28} + \frac{5x+8}{x^2+5x-14} \\
 = \frac{x+5}{(x+4)(x+7)} + \frac{5x+8}{(x+7)(x-2)} \\
 = \frac{x+5}{(x+4)(x+7)} \cdot \frac{x-2}{x-2} + \frac{5x+8}{(x+7)(x-2)} \cdot \frac{x+4}{x+4} \\
 = \frac{x^2+3x-10}{(x+4)(x+7)(x-2)} + \frac{5x^2+28x+32}{(x+4)(x+7)(x-2)} \\
 = \frac{6x^2+31x+22}{(x+4)(x+7)(x-2)}
 \end{aligned}$$

Solve the equation.

$$30) \frac{10}{a-10} = \frac{a}{a-10} - 3$$

$$\text{Let: } (a-10), a \neq 10$$

$$(a-10) \cdot \frac{10}{a-10} = (a-10) \cdot \frac{a}{a-10} - 3(a-10)$$

$$10 = a - 3a + 30$$

$$10 = -2a + 30$$

$$10-30 = -2a$$

$$-20 = -2a$$

$$10 = a$$

$\{ \}$ or NO Solution.