Math 54 **Test #2** **(Ch 4, 5, and 6)** **Name** Yuka Otani

98

Show all work neatly and systematically for full credit. Total points: 101

Find the indicated probability by using the special addition rule.

1) (6) A card is drawn from a well-shuffled deck of 52 cards.

a. What is the probability of drawing an ace **or** a 7?

$P(\text{an ace or a } 7) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$ ✓

b. What is the probability of drawing a spade or a queen.?

$P(\text{a spade or a queen}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$ ✓

2) (4 points)

a. You are dealt three cards successively **(without replacement)** from a shuffled deck of 52 playing cards. Find the probability that all cards are black.

$P(\text{all cards are black}) = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = 0.1176$ ✓

b. You are dealt three cards successively **(with replacement)** from a shuffled deck of 52 playing cards. Find the probability that all cards are black.

$P(\text{all cards are black}) = \frac{26}{52} \times \frac{26}{52} \times \frac{26}{52} = 0.125$ ✓

3) (6) The random variable X is the number of golf balls ordered by customers at a pro shop. Given the table of probabilities for X.

x	3	6	9	12	15
P(X = x)	0.14	0.21	0.36	0.19	0.10

a. Does this form a proability distribution? Explain.

Yes. Because each values of probability of X is less than 1 and the sum of all values of probability of X is 1, so this is probability distribution

b. Find the mean and standard deviation of the random variable.

mean: $\mu = 8.7$ standard deviation : $\sigma = 3.4857$

d. Find the probability that number of golf balls ordered by customers is odd.

$P(X \text{ is odd}) = (0.14) + (0.36) + (0.10)$
$= 0.6$ ✓

4) (6) In one large city, 73% of all voters are Democrats.

a. If three voters are randomly selected for a survey, find the probability that all three Democrats.

P (all three Democrats) = (0.73)(0.73)(0.73)

= 0.3890

b. If three voters are randomly selected for a survey, find the probability that at least one of them is a Democrat.

P (at least one of them is a Democrat) = 1 − P (no Democrat)

= 1 − (0.27)(0.27)(0.27)

= 0.9803

Find the indicated probability.

5) (12) The table below shows the soft drink preferences of people in three age groups.

	cola	root beer	lemon – lime	Total
under 21 years of age	40	25	10	75
between 21 and 40	35	20	25	80
over 40 years of age	20	30	35	85
Total	95	75	70	240

a. If one person is randomly selected, find the probability that the person is over 40 years of age.

P (person is over 40 years of age) $= \frac{85}{240} = \frac{17}{48}$

b. If one person is randomly selected, find the probability that the person is over 40 and drinks cola.

P (person is over 40 and drinks cola) $= \frac{20}{240} = \frac{1}{12}$

c. If one person is randomly selected, find the probability that the person is over 40 or drinks cola.

P (the person is over 40 or drinks cola) $= \frac{85}{240} + \frac{95}{240} - \frac{20}{240}$

$= \frac{160}{240} = \frac{2}{3}$

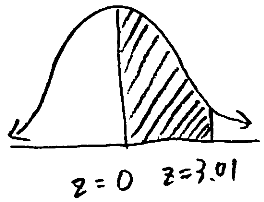
d. If three people are randomly selected, find the probability that all of them are over 40.

P (three people are over 40) $= \left(\frac{17}{48}\right)^3$

= 0.0444

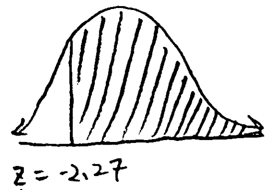
(8) Draw the density curve and use a table of areas to find the specified area under the standard normal curve.

6) a. Find P(0 < z < 3.01)



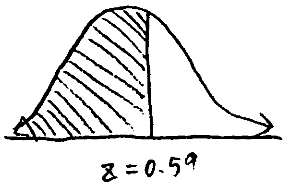
$P(0 < z < 3.01) = 0.9987 - 0.5$
$= 0.4987$

b. Find P(z > -2.27)



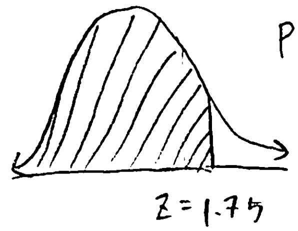
$P(z > -2.27)$
$= 0.9884$

c. Find P(z < 0.59)



$P(z < 0.59) = 0.7224$

d. Given P(z < a) = 0.96, find a.



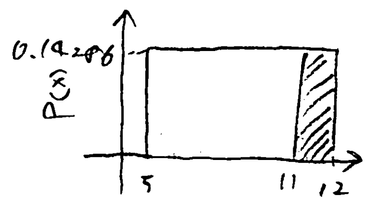
$P(z < a) = 0.96$

$a = 1.75$

7) (6) Assume that the weight loss for the first month of a diet program varies between 5 pounds and 12 pounds, and is spread evenly over the range of possibilities, so that there is **a uniform distribution.**

a. Find the probability of a random person lost more than 11 pounds

$P(X) \cdot 7 = 1$
$P(X) = 0.14286$
$P(X > 11) = 0.14286$



b. Find the proability of a random person lost between 8.5 pounds and 10 pounds.

$P(8.5 < X < 10) = 0.14286 \cdot (10 - 8.5)$
$= 0.21429$

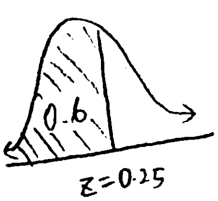
(4) Solve the problem. Round to the nearest tenth unless indicated otherwise.

8) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. Find P_{60}, the score which separates the lower 60% from the top 40%.

Normal
$\mu = 200$
$\sigma = 50$

$Z = 0.25$

$P_{60} = \mu + Z\sigma$
$= 200 + (0.25)\,50$
$= 212.5$



9) (12) A company manufactures calculators, and there is a 4% rate of defects. A batch of 50 calculators are randomly selected.

a. Find the probability of getting exactly 5 defects in a batch.

$P(X=5) = \text{binompdf}(50, 0.04, 5)$

$= 0.0346$ ✓

b. Find the probability of getting at most 4 defects in a batch.

$P(X \le 4) = \text{binomcdf}(50, 0.04, 4)$

$= 0.9510$ ✓

c. Find the probability of getting at least 3 defects in a batch.

$P(X \ge 3) = 1 - P(X < 3)$

$= 1 - P(X \le 2)$

$= 1 - \text{binomcdf}(50, 0.04, 2)$

$= 0.3233$ ✓

d. Find the mean and standard deviation of the number of defective calculators. Would it be unusual to have 10 defective calculators in a batch? Explain.

$\mu = np = 50(0.04) = 2$ ✓

$\sigma = \sqrt{npq} = \sqrt{50(0.04)(0.96)} = 1.3856$ ✓

$P(X = 10) = \text{binompdf}(50, 0.04, 10)$

$= 0.0000210$

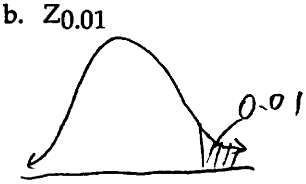
The probability of 10 defects in a batch is 0.0000210, so it would be unusual.

(4) Find the indicated value.

10) a. $z_{0.005}$

$z_{0.005} = 2.575$ ✓

b. $z_{0.01}$



$z_{0.01} = 2.33$ ✓

(8) Find the indicated probability.

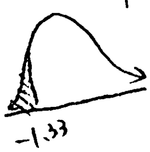
11) The incomes of trainees at a local mill are normally distributed with a mean of \$1100 and a standard deviation of \$150.

a. What percentage of trainees earn less than \$900 a month?

$$P(X < 900) = P\left(Z < \frac{900-1100}{150}\right)$$

$\mu = 1100$

$\sigma = 150$



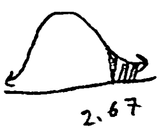
$$= P(Z < -1.33)$$

$$= 0.0918$$

$\therefore$ 9.18%

b. What percentage of trainees earn more than \$1500 a month?

$$P(X > 1500) = P\left(Z > \frac{1500-1100}{150}\right)$$



$$= P(Z > 2.67)$$

$$= 0.0038$$

$\therefore$ 0.38%

Solve the problem.

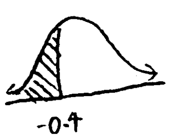
12) **(8)** The weights of the fish in a certain lake are normally distributed with a mean of 19 lb and a standard deviation of 6.

a. If a fish is randomly slected, what is the probability that the weight will be less than 16.6 lb?

$$P(X < 16.6) = P\left(Z < \frac{16.6-19}{6}\right)$$

$\mu = 19$

$\sigma = 6$



$$= P(Z < -0.4)$$

$$= 0.3446$$

b. If 9 fishes are randomly selected, what is the probability that the mean weight will be less than 16.6 lb?

$n = 9$

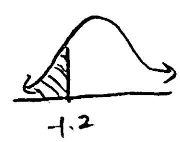
$\mu_{\bar{x}} = 19$

$\sigma_{\bar{x}} = \frac{6}{\sqrt{9}} = \frac{6}{3} = 2$

$$P(\bar{X} < 16.6) = P\left(Z < \frac{16.6-19}{2}\right)$$

$$= P(Z < -1.2)$$

$$= 0.1151$$



(4) Expected value.

13) Suppose you buy 1 ticket for \$1 out of a lottery of 1,000 tickets where the prize for the one winning ticket is to be \$500. What is your expected value?

	X (net profit)	P(x)
win	\$ 499	$\frac{1}{1000}$
lose	- \$ 1	$\frac{999}{1000}$

expected value : $\Sigma(X \cdot P(x))$

$$= 499\left(\frac{1}{1000}\right) - 1\left(\frac{999}{1000}\right)$$

$$= -0.5\$$$

$$= -50¢$$

Solve the problem.

14) **(4)** A final exam in Math 160 has a mean of 73 with standard deviation 7.8. If 44 students are randomly selected, find the probability that the mean of their test scores is greater than 78.

Normal

$n = 44$

$\mu_{\bar{x}} = 73$

$\sigma_{\bar{x}} = \frac{7.8}{\sqrt{44}}$

$= 1.176$

$P(\bar{x} > 78) = P\left(z > \frac{78-73}{1.176}\right)$

$= P(z > 4.25)$

$= 0.0001$

15) **(4)** In one county, the conviction rate for speeding is 85%. Use normal distribution as an approximation to the binomial to estimate the probability that of the next 100 speeding summonses issued, there will be at least 90 convictions.

$P(x \geq 90) = 1 - P(x \leq 89)$

$= 1 - \text{binomcdf}(100, 0.85, 89)$

$= 0.\cancel{9806}$

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(5) Provide an appropriate response.

16) Personal phone calls received in the last three days by a new employee were 3, 2, and 5. Assume that samples of size 2 are randomly selected with replacement from this population of three values. List the different possible samples, and find the mean of each of them. Then, construct a probability distribution of sample means.

the different possible sample

	$\bar{x}$
{3, 2}	2.5
{3, 3}	3
{3, 5}	4
{2, 3}	2.5
{2, 2}	2
{2, 5}	3.5
{5, 3}	4
{5, 2}	3.5
{5, 5}	5

a probability distribution of sample means

$\bar{x}$	$P(\bar{x})$
2	$\frac{1}{9}$
2.5	$\frac{2}{9}$
3	$\frac{1}{9}$
3.5	$\frac{2}{9}$
4	$\frac{2}{9}$
5	$\frac{1}{9}$

5