Math125 Test #2 (Chapter 9, 10) Name Yiyun Liang

98

Show all work neatly and systematically for full credit. Total points: 105 (3 points each, unless otherwise stated).

Solve. Write your solution in set builder notation or interval notation.

1) $8x + 2 \geq 4(3x + 1) - 18$

$8x + 2 \geq 12x + 4 - 18$

$-4x \geq -16$

$x \leq 4$

$\{x \mid x \leq 4\}$

Divide and, if possible, simplify. Assume all variables represent positive real numbers.

2) $\dfrac{\sqrt[3]{120x^4y^2}}{\sqrt[3]{15x^2y}}$

$= \sqrt[3]{\dfrac{120x^4y^2}{15x^2y}}$

$= \sqrt[3]{8x^2y}$

$= 2\sqrt[3]{x^2y}$

Rationalize the denominator. Assume all variables represent positive numbers.

3) $\sqrt[3]{\dfrac{7}{9x}}$

-1

$= \dfrac{\sqrt[3]{7}}{\sqrt[3]{9x}} \cdot \dfrac{\sqrt[3]{(9x)^2}}{\sqrt[3]{(9x)^2}}$

$= \dfrac{\sqrt[3]{567x^2}}{9x}$ → simplify.

Simplify by factoring.

4) $\sqrt[3]{135}$

$= \sqrt[3]{3^3 \cdot 5}$

$= 3\sqrt[3]{5}$

135
3 45
3 15
3 5

Multiply. Assume that all variables represent nonnegative real numbers.

5) $(3 + \sqrt{5})^2$

$= 3^2 + 2(3\sqrt{5}) + (\sqrt{5})^2$

$= 9 + 6\sqrt{5} + 5$

$= 14 + 6\sqrt{5}$

Add or subtract. Simplify by combining like radical terms, if possible. Assume all variables and radicands represent nonnegative numbers.

6) $\sqrt{5a} - 5\sqrt{80a} - 3\sqrt{20a}$

$= \sqrt{5a} - 20\sqrt{5a} - 6\sqrt{5a}$

$= -25\sqrt{5a}$

-1

Simplify.

7) $\sqrt[3]{729x^4y^5}$

$= 9xy\sqrt[3]{xy^2}$

729
3 243
3 81
9 9
3 3 3 3

-2

Solve the inequality and write the solution in either set builder notation or interval notation.

8) $-45 \le 7x - 10$ and $3x + 6 < 3$

$-45 + 10 \le 7x$ $\quad$ $3x < 3 - 6$

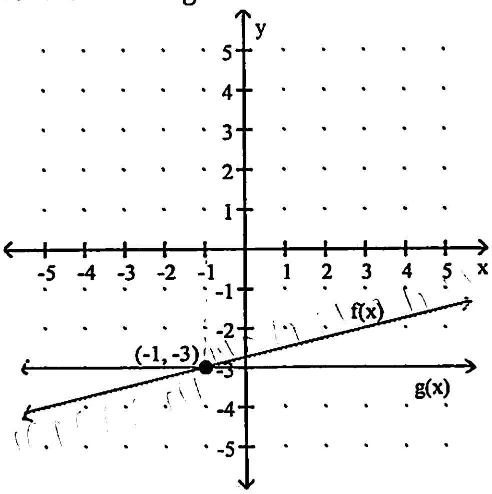
$-35 \le 7x$ $\quad$ $3x < -3$

$-5 \le x$ $\quad$ $x < -1$

$\{x \mid -5 \le x < -1\}$

Solve the inequality using the given graph.

9) Given the graph of f and g. write the solution for the following.



(a). Solve for f(x) < g(x).

-1

$(-\infty, -1]$

(b). Solve f(x) = g(x).

$\{-1\}$

(c). Solve f(x) > g(x).

$[-1, \infty)$

Add or subtract. Simplify by combining like radical terms, if possible. Assume all variables and radicands represent nonnegative numbers.

10) $4\sqrt[3]{27x} + 4\sqrt[3]{64x}$

$= 4 \cdot 3\sqrt[3]{x} + 4 \cdot 4\sqrt[3]{x}$

$= 12\sqrt[3]{x} + 16\sqrt[3]{x}$

$= 28\sqrt[3]{x}$

Multiply. Assume that all variables represent nonnegative real numbers.

11) $\sqrt{5}(\sqrt{3} - \sqrt{7})$

$= \sqrt{15} - \sqrt{35}$

Multiply and simplify. Assume all variables represent nonnegative real numbers. Write your answer in radical notation.

12) $\sqrt[4]{4a^2b}\sqrt{2ab^3c}$

$= 4^{\frac{1}{4}} a^{\frac{2}{4}} b^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} a^{\frac{1}{2}} b^{\frac{3}{2}} c^{\frac{1}{2}}$

$= 2^{\frac{1}{2}+\frac{1}{2}} a^{\frac{1}{2}+\frac{1}{2}} b^{\frac{1}{4}+\frac{3}{2}} c^{\frac{1}{2}}$

$= 2^{\frac{2}{2}} a^{\frac{2}{2}} b^{\frac{7}{4}} c^{\frac{1}{2}} = 2^{\frac{4}{4}} a^{\frac{4}{4}} b^{\frac{7}{4}} c^{\frac{2}{4}}$

$= 2ab\sqrt[4]{b^3c^2}$

Solve. Write your solution in set builder notation or interval notation.

13) $4x + 7(3x - 3) \le 9 - 5x$

$4x + 21x - 21 \le 9 - 5x$

$+5x \quad +21 \quad +21 \quad +5x$

$30x \le 30$

$x < 1$

$\{x \mid x < 1\}$

-1

Multiply. Assume that all variables represent nonnegative real numbers.

14) $(10\sqrt{2} + 10\sqrt{5})(5\sqrt{2} + 5\sqrt{5})$

$= 50\sqrt{4} + 50\sqrt{10} + 50\sqrt{10} + 50\sqrt{25}$

$= 100 + 100\sqrt{10} + 250$

$= 350 + 100\sqrt{10}$

Rationalize the denominator. Assume all variables represent positive numbers.

15) $\dfrac{\sqrt{3}}{\sqrt{5}+6}$

$= \dfrac{\sqrt{3}}{\sqrt{5}+6} \cdot \dfrac{\sqrt{5}-6}{\sqrt{5}-6}$

$= \dfrac{\sqrt{15} - 6\sqrt{3}}{(\sqrt{5})^2 - (6)^2}$

$= \dfrac{\sqrt{15} - 6\sqrt{3}}{5 - 36}$

$= \dfrac{\sqrt{15} - 6\sqrt{3}}{-31}$

Solve the equation.

16) $4|6x - 5| - 8 = -6$

$\frac{4|6x-5|}{4} = \frac{2}{4}$

$|6x - 5| = \frac{1}{2}$

$6x - 5 = \frac{1}{2}$ or $6x - 5 = -\frac{1}{2}$

$6x = 5\frac{1}{2}$ $\quad$ $6x = 4\frac{1}{2}$

$x = \frac{11}{12}$ $\quad$ $x = \frac{3}{4}$

$\{\frac{11}{12}, \frac{3}{4}\}$

Solve the inequality. Write the solution in either set builder notation or interval notation.

17) $-5x + 1 \geq 11$ or $6x + 3 \geq -21$

$-5x \geq 10$ $\quad$ $6x \geq -24$

$x \leq 2$ $\quad$ $x \geq -4$

$\mathbb{R}$

(5) **Solve the equation.**

18) $\left|\frac{1}{2}n + 2\right| = \left|\frac{3}{4}n - 2\right|$

$4(\frac{1}{2}n + 2 = \frac{3}{4}n - 2)$ or $\frac{1}{2}n + 2 = -(\frac{3}{4}n - 2)$

$2n + 8 = 3n - 8$ $\quad$ $4(\frac{1}{2}n + 2 = -\frac{3}{4}n + 2)$

$-n = -16$ $\quad$ $2n + 8 = -3n + 8$

$n = 16$ $\quad$ $5n = 0$

$n = 0$

$\{0, 16\}$

Solve.

19) $\sqrt[3]{x + 5} - 2 = 0$

$(\sqrt[3]{x+5})^3 = (2)^3$

$x + 5 = 8$

$x = 3$

$\{3\}$

-0.5

Solve the inequality.

20) Company A rents copiers for a monthly charge of \$250 plus 10 cents per copy. Company B rents copiers for a monthly charge of \$500 plus 5 cents per copy. What is the number of copies above which Company A's charges are the higher of the two?

$250 + 0.10x > 500 + 0.05x$ ✓

$0.05x > 250$ ✓

$x > 5000$ ✓

For more than 5000 copies that Company A's charges are the higher of the two.

Graph the system of linear inequalities.

21) $x - 2y \le 2$ and $x + y < 0$

$x - 2y \le 2$

$-2y \le 2 - x$

$y \ge -1 + \frac{1}{2}x$

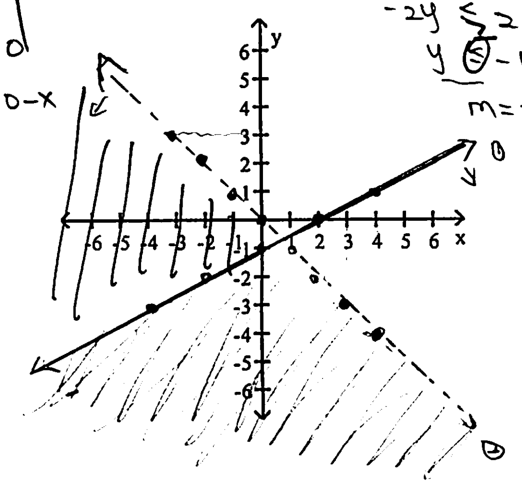
$m = \frac{1}{2}$ $(0, -1)$

$x + y < 0$

$y < 0 - x$

$m = 1$

$(0, 0)$



Divide and, if possible, simplify. Assume all variables represent positive real numbers.

22) $\dfrac{\sqrt[4]{x^2y^3}}{\sqrt[3]{xy}}$

$= \dfrac{(x^2y^3)^{\frac{1}{4}}}{(xy)^{\frac{1}{3}}}$

$= \dfrac{x^{\frac{1}{2}}y^{\frac{3}{4}}}{x^{\frac{1}{3}}y^{\frac{1}{3}}}$

$= x^{\frac{1}{2}-\frac{1}{3}}y^{\frac{3}{4}-\frac{1}{3}}$

$= x^{\frac{3}{6}-\frac{2}{6}}y^{\frac{9}{12}-\frac{4}{12}}$

$= x^{\frac{1}{6}}y^{\frac{5}{12}}$

$= x^{\frac{2}{12}}y^{\frac{5}{12}}$

$= \sqrt[12]{x^2y^5}$ ✓

Write the domain of f in interval notation or interval notation.

23) (a). $f(x) = \dfrac{x+1}{7x-9}$

$7x - 9 = 0$

$7x = 9$

$x = \frac{9}{7} = 1\frac{2}{7}$

$\{x \mid x \text{ is real number, and } x \ne \frac{9}{7}\}$

(b). $f(x) = \sqrt{1 + 7x}$

$1 + 7x \ge 0$

$7x \ge -1$

$x \ge -\frac{1}{7}$

$\{x \mid x \ge -\frac{1}{7}\}$ ✓

Solve the absolute-value inequality. Write the slution in interval notation or set builder notation.

24) $|b + 7| - 6 > 9$

$|b+7| > 15$

$b + 7 > 15$ or $b + 7 < -15$

$b > 8$ $\quad$ $b < -22$

$\{b \mid b > 8 \text{ or } b < -22\}$ ✓

-1

Solve and write the solution in either set builder notation or interval notation.

25) $|3k + 2| + 9 < 18$

$|3k+2| < 9$

$-9 < 3k + 2 < 9$

$-11 < 3k < 7$

$-\frac{11}{3} < k < \frac{7}{3}$

$-3\frac{2}{3} < k < 2\frac{1}{3}$

$\{k \mid -3\frac{2}{3} < k < 2\frac{1}{3}\}$

Solve the equation.

26) $|8m + 3| + 6 = 14$

$|8m+3| = 8$

$8m+3 = 8$ or $8m+3 = -8$

$8m = 5$ $\quad$ $8m = -11$

$m = \frac{5}{8}$ $\quad$ $m = -\frac{11}{8}$

$m = -1\frac{3}{8}$

$\{\frac{5}{8}, -1\frac{3}{8}\}$

Simplify by taking the roots of the numerator and the denominator. Assume all variables represent positive numbers.

27) $\sqrt[3]{\frac{1296x^4}{6x}}$

216
6 36
6 6

$= \sqrt[3]{216x^3}$

$= \sqrt[3]{6^3 \cdot x^3}$

$= 6x$

Simplify. Assume that no radicands were formed by raising negative numbers to even powers.

28) $\sqrt[5]{x^{19}y^7z^{12}}$

$= x^3 y z^2 \sqrt{x^4 y^2 z^2}$

Simplify by taking the roots of the numerator and the denominator. Assume all variables represent positive numbers.

29) $\sqrt{\frac{175a^2b}{c^2}}$

175
7 25
5 5

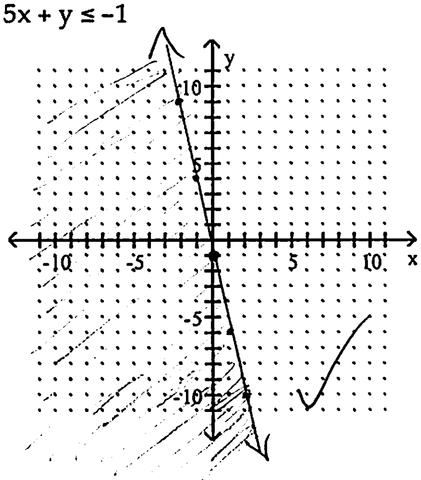
$= \frac{5a\sqrt{7b}}{c}$

Graph on a plane.

30) $5x + y \leq -1$

$y \leq -1 - 5x$

$m = \frac{-5}{1}$ $(0, -1)$



Multiply and simplify. Assume all variables represent nonnegative real numbers. Write your answer in radical notation.

31) $\sqrt{14m^5}\sqrt{7m^{13}}$

$= \sqrt{98m^{18}}$

$= 7m^9\sqrt{2}$ ✓

98
2 49
7 7

Use rational exponents to simplify. Do not use fraction exponents in the final answer. Assume that even roots are of nonnegative quantities.

32) $\sqrt[5]{x\sqrt[3]{x}}$

$= \sqrt{x}^{(3\cdot 5)}$

$= \sqrt[15]{x}$ ✗

-3

Correction:

$\sqrt[5]{x\sqrt[3]{x}}$

$= (x \cdot x^{\frac{1}{3}})^{\frac{1}{5}}$

$= x^{\frac{1}{5}} \cdot x^{\frac{1}{15}}$

$= x^{\frac{4}{15}}$

$= \sqrt[15]{x^4}$

Simplify. Assume that variables can represent any value.

33) a. $\sqrt{36x^2}$

$= |6x|$ ✓

b. $\sqrt{25x^2 + 30x + 9}$

$= |(5x + 3)|$ ✓

(6) Solve.

34) $\sqrt{3x+1} = 3 + \sqrt{x-4}$

$(\sqrt{3x+1})^2 = (3 + \sqrt{x-4})^2$

$3x + 1 = 9 + 6\sqrt{x-4} + x - 4$

$3x + 1 - 9 + 4 - x = 6\sqrt{x-4}$

$\frac{2x - 4}{2} = \frac{6\sqrt{x-4}}{2}$

$(x - 2)^2 = (3\sqrt{x-4})^2$

$x^2 - 2(2x) + 2^2 = 9(x-4)$

$x^2 - 4x + 4 = 9x - 36$

$x^2 - 13x + 40 = 0$

$(x-5)(x-8) = 0$

$x = 5 \quad x = 8$ ✓

40
10 4

Check: $\sqrt{3\cdot 5 + 1} = 3 + \sqrt{5-4}$

$\sqrt{16} = 3 + \sqrt{1}$

$4 = 3 + 1$ True

$\sqrt{3\cdot 8 + 1} = 3 + \sqrt{8-4}$

$\sqrt{25} = 3 + \sqrt{4}$

$5 = 3 + 2$ True

$\{5, 8\}$ ✓

-3