Math 227 Test #3 (Ch 4, 5, 6) Name Jiamin Ruan

102

Show all work neatly and systematically for full credit. Total points: 105

Find the indicated probability by using the special addition rule.

1) (4) The Math Club at ELAC has 15 members.

a. If the club must appoint a planning committee with four different members, how many ways can a committee be appointed?

$_{15}C_4 = 1365$

b. If the club must appoint a president, a vice president, a secretary, and a treasurer, how many different ways can the four officers be appointed?

$_{15}P_4 = 32760$

2) (10) In a sandwich shop, the following probability distribution was obtained. The random variable x represents the number of condiments used for a hamburger.

x	P(x)	$x \cdot p(x)$	$x^2 p(x)$
0	0.30	0	0
1	0.40	0.4	0.4
2	0.20	0.4	0.8
3	0.06	0.18	0.54
4	0.04	0.16	0.64
		1.14	2.38

a. Verify that the above is a discrete probability distribution.

1) x is discrete random variable

2) $\Sigma P(x) = 1$

3) $0 \le P(x) \le 1$

yes, it's a probability dist.

b. Find the mean for the random variable x.

mean : $\Sigma x\, p(x) = 1.14$

d. Find the standard deviation for the random variable x.

$\sigma = \sqrt{2.38 - 1.14^2} = 1.039$

e. Find the probability that a sandwich shop uses at most 1 condiment.

$P(x \le 1) = 0.4 + 0.3 = 0.7$

f. A sandwish shop used 4 condiments for its hamburgers, would it be a significantly high number of condiments? Explain.

$P(4 \text{ or more}) = P(x \ge 4) = 0.04 < 0.05$

$\mu + 2\sigma$
$= 1.14 + 2(1.039)$
$= 3.218$

yes, a sandwish shop used 4 condiment is a significantly high number.

3) (9) Sixty-two percent of the people that use the Internet order something online. 15 Internet user are randomly selected.

62%

Binomial

a. Find the probability that 5 of 15 Internet users will order something online.

$p = 0.62$
$q = 0.38$
$n = 15$
$x = 5$

$P(X=5) = \text{Binompdf}(15, 0.62, 5) = 0.0173$

b. Find the probability that at least 4 of 15 Internet users will order something online.

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \text{Binomcdf}(15, 0.62, 3)$
$= 0.999$

c. Find the probability that at most 10 of 15 Internet users will order something online.

$P(X \leq 10) = \text{Binomcdf}(15, 0.62, 10) = 0.733$

4) (6) A quiz consists of 50 multiple choice questions, each with five possible answers, only one of which is correct.

a. If a student guesses on each question, what is the mean and standard deviation of the number of correct answers?

$p_{\text{correct}} = \frac{1}{5}$
$q_{\text{wrong}} = \frac{4}{5}$
$n = 50$

$\mu = np = (50)(\frac{1}{5}) = 10$
$\sigma = \sqrt{npq} = \sqrt{(50)(\frac{1}{5})(\frac{4}{5})} = 2.828$

b. If a student guesses on each qustion, would getting 35 of 50 correct answers be significantly hgih number of correct answers? Explain.

$\mu + 2\sigma = 10 + 2(2.828)$
$= 15.656$

$P(35 \text{ or more}) = P(x \geq 35) = 1 - P(x \leq 34)$
$= 1 - \text{Binomcdf}(50, \frac{1}{5}, 34)$
$= 0 < 0.05$

yes, getting 35 of 50 correct answer is significantly high number

5) (4) Investing is a game of chance. Suppose there is a 34% chance that a risky stock investment will end up in a total loss of your investment. Because the rewards are so high, you decide to invest four independent risky stocks. Find the probability that at least one of your four investments becomes a total loss.

34% total loss

66% total win

$P(\text{at least 1 of 4 total loss})$
$= 1 - P(\text{4 total win})$
$= 1 - (66\%)^4$
$= 0.810$

Find the indicated probability. Give your answer as a simplified fraction.

6) (12) The manager of a used car lot took inventory of the automobiles on his lot and constructed the following table based on the age of his car and its make (foreign or domestic).

	Age of Car (in years)				
Make	0 – 2	3 – 5	6 – 10	over 10	Total
Foreign	45	29	12	14	100
Domestic	36	28	15	21	100
Total	81	57	27	35	200

a. A car was randomly selected from the lot. Given that the car selected was a foreign car, what is the probability that it was older than 2 years?

$P(\text{older than } 2 \mid \text{foreign car}) = \dfrac{29+12+14}{100} = 0.55$

b. A car was randomly selected from the lot, find the probability that it was a foreign car.

$P(\text{foreign car}) = \dfrac{100}{200} \quad 0.5$

c. A car was randomly selected from the lot, find the probability that it was a foreign car or it was older than 10 years.

$P(\text{foreign car or older than } 10)$

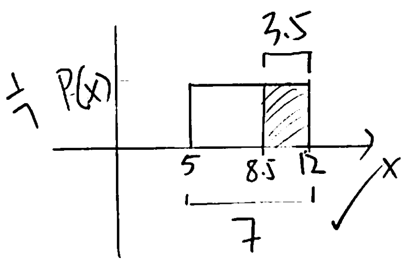
$= \dfrac{100}{200} + \dfrac{35}{200} - \dfrac{14}{200}$

$= 0.605$

d. Three cars were randomly selected from the lot, find the probability that they are all domestic cars.

$P(3 \text{ domestic car}) = \dfrac{100}{200} \cdot \dfrac{99}{199} \cdot \dfrac{98}{198} = 0.123$

7) (4) Assume that the weight loss for the first month of a diet program varies between 5 pounds and 12 pounds, and is spread evenly over the range of possibilities, so that there is a uniform distribution. Find the probability of a random person loss more than 8.5 pounds

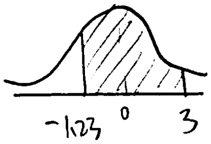


$P(X>8.5) = (12-8.5)\left(\frac{1}{7}\right)$

$= 0.5$

(8) **Draw the density curve and use a table of areas to find the specified area under the standard normal curve.**

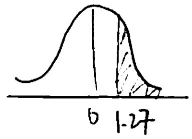
8) a. Find P(−1.23 < z < 3)



$= 0.9987 - 0.1093$

$= 0.8894$ ✓

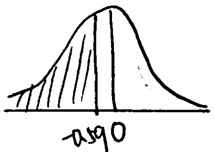
b. Find P(z > 1.27)



$= P(Z < -1.27)$

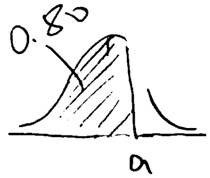
$= 0.1020$ ✓

c. Find P(z < − 0.59)



$= 0.2776$ ✓

d. Given P(z < a) = 0.80, find a.



$a = 0.84$ ✓

$Z < 0.84$

9) (4) Suppose you pay $3.00 to roll a fair die with the understanding that you will get back $5.00 for rolling a 1 or a 6, nothing otherwise. What is your expected value?

$5 - 3 = 2$

	X	P(X)
loss	$ −3	$\frac{4}{6}$
win	$ 2	$\frac{2}{6}$

$E(x) = (-3)(\frac{4}{6}) + (2)(\frac{2}{6})$ ✓

$= -1.33$ ✓

expected value : $ ~~−~~1.33

10) (6) The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 6.4.

a. Find the probability that exactly three accidents will occur next month on this stretch of road.

$\mu = 6.4$ ✓

$P(X = 3) =$ poissonpdf (6.4, 3)

$= 0.0726$ ✓

b. Find the probability that at least two accidents will occur next month on this stretch of road.

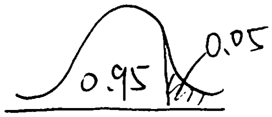
$P(X \geq 2) = 1 - P(X \leq 1)$ ✓

$= 1 -$ poisson cdf (6.4, 1) ✓

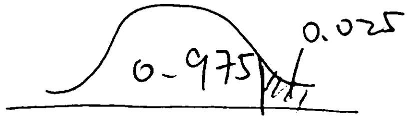
$= 0.988$ ✓

(4) Find the indicated value.

11) a. $z_{0.05}$



b. $z_{0.025}$



$Z_{0.05} = 1.645$ ✓

$Z_{0.025} = 1.96$ ✓

(4) Counting./probability

12) a. Winning the jackpot of the Mega Millions lottery requires that you select the correct five different number between 1 and 70 and, in a separate drawing, you must also select the correct single numbet between 1 and 25.

a. How many possible Mega Millions lottery tickets?

$\underline{70}\ \underline{70}\ \underline{70}\ \underline{70}\ \underline{70}\ \underline{25} = \cancel{\times} 4.202 \times 10^{10}$

b. What is the proability of winning the jackpot?

-3

$P(\text{wining}) = \dfrac{1}{4.202\cancel{\times}10^{10}}$

(9) Probability.

13) a. Find the probability that a randomly selected student has birthday on April 1.

$P(\text{birthday on Apr } 1) = \dfrac{1}{365}$ ✓

b. Find the probability that of 5 randomly selected students, no two share the same birthday.

$P(\text{diff birthday}) = \dfrac{365}{365} \cdot \dfrac{364}{365} \cdot \dfrac{363}{365} \cdot \dfrac{362}{365} \cdot \dfrac{361}{365} = 0.973$ ✓

c. Find the probability that 5 randomly selected students have same birthday.

$P(\text{same birth}) = \dfrac{365}{365} \cdot \dfrac{1}{365} \cdot \dfrac{1}{365} \cdot \dfrac{1}{365} \cdot \dfrac{1}{365} = 5.63 \times 10^{-11}$ ✓

14) (4) The length of time it takes college students to find a parking spot in the library parking lot follows a normal distribution with a mean of 7 minutes and a standard deviation of 1.2 minute. Find the probability that a randomly selected college student will find a parking spot in the library parking lot in less than 6.0 minutes.

$\mu = 7$
$\sigma = 1.2$

$P(x < 6) = P(Z < \frac{6-7}{1.2})$
$= P(Z < -0.83)$
$= 0.2033$ ✓

-3

15) (3, 4) Assume that the heights of men are normally distributed with a mean of 69.8 inches and a standard deviation of 2.8 inches.

a. If a man is randomly selected, find the probability that he has height greater than 65 inches.

$\mu = 69.8$
$\sigma = 2.8$

$P(X>65) = P(Z > \frac{65-69.8}{2.8}) = P(Z > -1.71)$
$= P(Z < 1.71)$
$= 0.9564$ ✓

b. If 64 men are randomly selected, find the probability that they have a mean height greater than 70.8 inches.

$n = 64$
$\mu_{\bar{x}} = 69.8$
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.8}{\sqrt{64}} = 0.35$

$P(\bar{X} > 70.8) = P(Z > \frac{70.8 - 69.8}{0.35})$
$= P(Z > 2.86)$
$= P(Z < -2.86)$ ✓
$= 0.0021$

16) (4) Assume that blood pressure readings are normally distributed with a mean of 117 and a standard deviation of 9.6. If 144 people are randomly selected, find the probability that their mean blood pressure will be less than 119.

$\mu = 117$
$\sigma = 9.6$
$n = 144$
$\sigma_{\bar{x}} = \frac{9.6}{\sqrt{144}} = 0.8$

$P(\bar{X} < 119) = P(Z < \frac{119-117}{0.8})$
$= P(Z < 2.5)$
$= 0.9938$ ✓

17) (3, 3) Personal phone calls received in the last three days by a new employee were 3, 2, and 5. Assume that samples of size 2 are randomly selected with replacement from this population of three values.

a. List the different possible samples, and find the mean of each of them.

Sample	$\bar{x}$
3, 3	3
3, 2	2.5
3, 5	4
2, 3	2.5
2, 2	2
2, 5	3.5
5, 3	4
5, 2	3.5
5, 5	5

✓

$\{3, 2, 5\}$
$\mu = 3.33$
$\sigma = 1.25$

b. Construct a sampling distribution of sample means. Then find the mean and the standard deviation of the probability distribution.

$\bar{x}$	$P(\bar{x})$
2	1/9 = 0.111
2.5	2/9 = 0.222
3	1/9 = 0.111
3.5	2/9 = 0.222
4	2/9 = 0.222
5	1/9 = 0.111

✓

$\mu = \sum(\bar{x} \cdot P(\bar{x})) = 3.33$

$\sigma = \frac{1.25}{\sqrt{2}} = 0.884$ ✓

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