Math227 **Test #3** **Name** Jiamin Ruan

97

Show all work neatly and systematically for full credit. Total points:100.
Note: for hypothesis testing and confidence interval, make sure to show all steps. Remember to write conclusion sentences.

1) (7) Leakage from underground fuel tanks has been a source of water pollution. In a random sample of 107 gasoline stations, 18 were found to have at least one leaking underground tank. Construct a 95% confidence interval for the proportion of gasoline stations with at least one leaking underground tank.

95% CI ✓

$n = 107$
$x = 18$
$\hat{p} = \frac{18}{107} = 0.168$
$\hat{q} = 0.832$

$n\hat{p} = 17.976 > 5$
$n\hat{q} = 89.024 > 5$ ⇒ binomial

① $z_{\alpha/2} = 1.96$

② $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96 \cdot \sqrt{\frac{(0.168)(0.832)}{107}} = 0.0708$ ✓

③ $\hat{p} - E < p < \hat{p} + E$

$0.0972 < p < 0.2388$ ✓

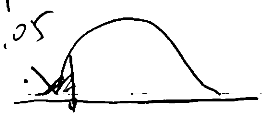
we're 95% confident that the interval from 0.0972 to 0.2388 contain the proportion of gasoline station with at least one leaking underground tank

(8) Find the critical value(s).

2) a. Determine the critical value. Assume t distribution, it is a left-tailed test of a population mean at the $\alpha = 0.05$ level of significance and $n = 15$.

-1

0.05



$\alpha = 0.05$, $df = 14$, one tail.

$t_{critical}$ -1.761 ✓

b. Determine the critical value. Assume normal distribution. Test the claim about the population proportion $p > 0.28$, and $\alpha = 0.01$.

0.01

$z = 2.33$ ✓

c. Determine the critical values for a two-tailed test of a population standard deviation for a sample of size $n = 40$ at the $\alpha = 0.01$ level of significance. Assume Chi-square distribution.

$\alpha/2$

$\alpha/2$ 0.005

df=39

$\chi^2_R = 66.766$ ✓

$\chi^2_L = 20.707$ ✓

d. Determine the critical value. Assume normal distribution, it is a left-tailed test of a population proportion at the $\alpha = 0.10$ level of significance and $n = 150$.

-2

0.10

df = 149

t -1.290

$z = -1.28$

-3

3) (4, 4) A research wantsto estimate the proportion of households that have broadbank Internet access. What size sample should be obtained if she wishes the estimate to be within two percentage point and with 99% confidence if

a. she uses an estimate of 0.635 obtained from the National Telecommunications and Information Administration?

99% CI $Z_{\alpha/2} = 2.575$

$E = 0.02$

$\hat{p} = 0.635$ $\hat{q} = 0.365$

$$n = \frac{Z_{\alpha/2}^2 \hat{p}\hat{q}}{E^2} = \frac{2.575^2 (0.635)(0.365)}{0.02^2}$$

$= 3842.03$

≈ 3843

b. she does not use any prior estimate?

$$n = \frac{2.575^2 (0.5)(0.5)}{0.02^2} = 4144.14$$

≈ 4145

4) (6) The principal at Riverside High School would like to estimate the mean length of time each day that it takes all the buses to arrive and unload the students. How large a sample is needed if the principal would like to assert with 96% confidence that the sample mean is within 5 minutes? Assume that s = 10 minutes based on previous studies.

96% CI

$Z_{\alpha/2} = 2.05$

$$n = \left(\frac{2.05 \cdot 10}{5}\right)^2 = 16.81 \approx 17$$

$$n = \left(\frac{Z_{\alpha/2}\, \sigma}{E}\right)^2$$

5) (8) The football coach randomly selected ten players and timed how long each player took to perform a certain drill. The times (in minutes) were:

7.2 10.5 9.9 8.2 11.0

7.3 6.7 11.0 10.8 12.4

Determine a 95% confidence interval for the mean time for all players. Assume that the population has a normal distribution.

$n = 10$

$\bar{x} = 9.5$

$s = 1.984$

σ is unknown $\Rightarrow$ t-dist

95% CI

2-tailed

df = 9

$t_{\frac{\alpha}{2}} = 2.262$

$$E = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} = 2.262 \cdot \frac{1.984}{\sqrt{10}} = 1.419$$

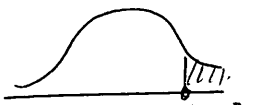
$\bar{x} - E < \mu < \bar{x} + E$

$8.081 < \mu < 10.919$

We are 95% confident that the interval from 8.081 to 10.919 contains the true mean time for all players

(8) **Use the given information to find the P-value.**

6) a. Assume normal distribution. The test statistic in a right-tailed test is $z = 1.02$. Find p-value.



$z = 1.02$

P-value = normal cdf (1.02, E99) = 0.1539 ✓

b. Assume t-distribution with $n = 45$. It is a two-tailed test. with test statistic $t = -1.635$. Find p-value

P-value = tcdf (−E99, −1.635, 44) X2 = 0.1092 ✓

c. Assume chi-square distribution with $n = 14$. It is a left-tailed test and the test statistic is $X^2 = 6.278$. Find p-value.

P-value = X_L^2 cdf (0, 6.278, 13) = 0.06455 ✓

d. Assume normal distribution. The test statistic is $z = 3.01$, and it is a two-tailed test. Find P-value.

p value = normal cdf (3.01, E99) X 2 = 0.002613 ✓

7) (8) A poll of 1068 adult Americans reveals that 52% of the voters surveyed prefer the Democratic candidate for the presidency. At the 0.05 level of significance, test the claim that more than half of all voters prefer the Democrat.

① Claim : $p > 0.5$

$H_0 : p = 0.5$ ✓

$H_1 : p > 0.5$

② $n = 1068$

$\hat{p} = 0.52$ ✓

$np = 534 > 5$

$nq = 534 > 5$

⇒ normal dist

③ $z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \dfrac{0.52 - 0.5}{\sqrt{\frac{0.25}{1068}}} = 1.307$ ✓

④ P-value = normalcdf (1.307, E99) = 0.0956 ✓

⑤ P-value > α fail to reject H_0 ✓

(0.0956) (0.05) ✓

There is not sufficient evidence to support the claim that more than half of all voters prefer the democrat.

8) (8) A public bus company official claims that the mean waiting time for bus number 14 during peak hours is less than 10 minutes. Karen took bus number 14 during peak hours on 18 different occasions. Her mean waiting time was 7.5 minutes with a standard deviation of 1.7 minutes. At the 0.01 significance level, test the claim that the mean waiting time is less than 10 minutes. Assume that sample is drawn from normally distributed population.

① claim: $\mu < 10$

$H_0: \mu = 10$

$H_1: \mu < 10$ ✓

② $n = 18$

$\bar{x} = 7.5$

$s = 1.7$

normal distributed

$\Rightarrow$ t-dist. ✓

③ $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{7.5 - 10}{\frac{1.7}{\sqrt{18}}} = -6.239$ ✓

④ $\alpha = 0.01$, left tail, $df = 17$

p-value = tcdf $(-E99, -6.239, 17) = 4.5 \times 10^{-6}$ ✓

⑤ p-value $< \alpha$ reject H_0 $t_{crit} = -2.567$

(4.5×10^{-6}) (0.01) ✓

There's sufficient evidence to support the claim that the mean waiting time is less than 10 min. ✓

9) (8) In a sample of 87 young adult, the average time per day spent in bed asleep was 7.06 hours and the standard deviation was 1.11 hours. Construct a 99% confidence interval for the mean time spent in bed asleep.

$n = 87$

$\bar{x} = 7.06$

$s = 1.11$

$n > 30$

σ is unknow

$\Rightarrow$ t-dist.

① 99% CI, $df = 86$, 2 tails

$t_{\frac{\alpha}{2}} = 2.632$ ✓

② $E = t_{\frac{\alpha}{2}} \cdot \dfrac{s}{\sqrt{n}} = 2.632 \cdot \dfrac{1.11}{\sqrt{87}} = 0.313$ ✓

③ $\bar{x} - E < \mu < \bar{x} + E$

$6.747 < \mu < 7.373$

We are 99% confident that the interval from 6.747 to 7.373 contain the mean time spent in bed asleep.

10) (8) For randomly selected adults, IQ scores are normally distributed with a standard deviation of 15. The scores of 14 randomly selected college students are listed below. Use a 0.10 significance level to test the claim that the standard deviation of IQ scores of college students is less than 15.

115	128	107	109	116	124	135
127	115	104	118	126	129	133

① claim : $\sigma < 15$

$H_0 : \sigma = 15$

$H_1 : \sigma < 15$ ✓

② $n = 14$

$s = 9.819$

normal distributed
$\Rightarrow \chi^2$ dist ✓

③ $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(13)(9.819)^2}{15^2} = 5.5705$ ✓

④ Left tail, df = 13 ✓

p-value = χ^2_i cdf (0, 5.5705, 13) = 0.03960 ✓

⑤ p-value < α reject H_0 ✓ , $\chi^2_{crit} = 7.042$.
(0.03960). (0.10)

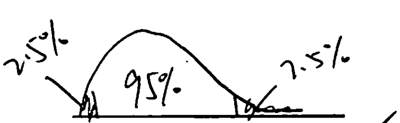
There is sufficient evidence to support the claim that the standard deviation of IQ scores of college students is less than 15. ✓

11) (8) A sociologist develops a test to measure attitudes about public transportation, and 27 randomly selected subjects are given the test. Their mean score is 76.2 and their standard deviation is 21.4. Construct the 95% confidence interval for the standard deviation, σ, of the scores of all subjects. Assume the population is normal.

$n = 27$

$s = 21.4$

95% CI



df=26 ✓

① $\chi^2_L = 13.844$ ✓

$\chi^2_R = 41.923$ ✓

② $\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$ ✓

$16.853 < \sigma < 29.327$ ✓

we are 95% confident that the interval from 16.853 to 29.327 contain the true standard deviation of the score of all subject. ✓

12) (7) Of 101 randomly selected adults, 35 were found to have high blood pressure. Construct a 95% confidence interval for the true percentage of all adults that have high blood pressure.

$n = 101$

$x = 35$

$\hat{p} = \frac{35}{101} = 0.347$

$\hat{q} = 0.653$

$n\hat{p} > 5$

$n\hat{q} > 5$

$\Rightarrow$ binomial

$z_{\alpha/2} = 1.96$

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\cdot\hat{q}}{n}} = 1.96\cdot\sqrt{\frac{(0.347)(0.653)}{101}} = 0.0928$

$\hat{p} - E < p < \hat{p} + E$

$0.2542 < p < 0.4398$

we are 95% confident that the interval from 0.2542 to 0.4398 contain the true percentage of all adults that have high blood pressure.

(8) Express the null hypothesis and the alternative hypothesis in symbolic form.

13) a. A researcher claims that the amounts of acetaminophen in a certain brand of cold tablets have a standard deviation different from 3.3 mg claimed by the manufacturer.

$H_0: \sigma = 3.3$

$H_1: \sigma \neq 3.3$

b. A psychologist claims that more than 4.1 percent of the population suffers from professional problems due to extreme shyness. Use p, the true percentage of the population that suffers from extreme shyness.

$H_0: p = 4.1\%$

$H_1: p > 4.1\%$

c. The owner of a football team claims that the average attendance at games is over 62,900, and he is therefore justified in moving the team to a city with a larger stadium.

$H_0: \mu = 62900$

$H_1: \mu > 62900$

d. A cereal company claims that the mean weight of the cereal in its packets is at least 14 oz.

$\geq$

$H_0: \mu \geq 14$

$H_1: \mu < 14$