Math 54 **Test #3 (Ch 4, 5, 6)** **Name** Solution

Show all work neatly and systematically for full credit. Total points: 100

1) (10) In a sandwich shop, the following probability distribution was obtained. The random variable x represents the number of condiments used for a hamburger.

x	P(x)	$x^2 \cdot P(x)$
0	0.30	0
1	0.40	0.4
2	0.20	0.8
3	0.06	0.54
4	0.04	0.64
		2.38

a. Verify that the above is a discrete probability distribution.

1) X is a discrete random variable.

2) $\sum P(x) = 1$

3) $0 \le P(x) \le 1$, for each X.

Hence, it is a probability distribution.

b. Find the mean for the random variable x.

$\mu = \sum X \cdot P(x)$

$= 0(0.3) + 1(0.4) + 2(.2) + 3(.06) + 4(.04) = \underline{\underline{1.14}}$.

d. Find the standard deviation for the random variable x.

$\sigma^2 = \sum X^2 \cdot P(x) - \mu^2$

$= 2.38 - 1.14^2$

$= 1.0804$

$\sigma = \sqrt{1.0804}$

$= 1.03942$.

e. Find the probability that a sandwich shop uses at most 1 condiment.

$P(X \le 1) = P(X=0) + P(X=1)$

$= 0.30 + 0.40$

$= 0.70$

f. A sandwish shop uses 4 condiments, would it be significantly high number of condiments? Explain.

$\mu + 2\sigma$

$= 1.14 + 2(1.03942)$

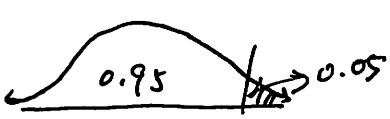
$= 3.21884$

$4 > \mu + 2\sigma$,

So, use of 4 Condiments would be significantly high number of Condiments.

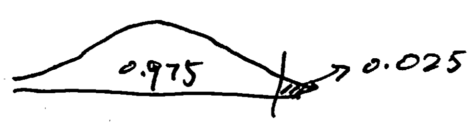
(6) Find the indicated value.

2) a. $z_{0.05}$



$z_{0.05} = 1.645$

b. $z_{0.025}$



$z_{0.025} = 1.96$

3) (12) Sixty-two percent of the people that use the Internet order something online. 15 Internet user are randomly selected.

a. Find the probability that 5 of 15 Internet users will order something online.

X: # of Internet user order something.

$p = 62\% = 0.62$

$q = 0.38$

$n = 15$

$P(X=5)$

$= \text{binompdf}(15, 0.62, 5)$

$= 0.0173$

b. Find the probability that at least 4 of 15 Internet users will order something online.

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - \text{binomcdf}(15, 0.62, 3)$

$= 0.9989$

c. Find the probability that at most 10 of 15 Internet users will order something online.

$P(X \leq 10) = \text{binomcdf}(15, 0.62, 10)$

$= 0.7334$

4) (8) A quiz consists of 50 multiple choice questions, each with five possible answers, only one of which is correct.

a. If a student guesses on each question, what is the mean and standard deviation of the number of correct answers?

X: # of correct answers

$n = 50$

$p = \frac{1}{5} = 0.2$

$q = 0.8.$

$\mu = np = 50(0.2) = 10$

$\sigma = \sqrt{n p \cdot q} = \sqrt{50(.2)(.8)} = 2.8284$

b. If a student guesses on each qustion, would getting 35 of 50 correct answers be significantly hgih number of correct answers? Explain.

$\mu + 2\sigma$

$= 10 + 2(2.8284)$

$= 15.6568$

Since $35 > \mu + 2\sigma$, it would be a significantly high number of correct answers.

5) (4) A card is drawn from a standard deck of 52 playing cards. Find the probability that the card is king or a heart.

$P(\text{king or heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

6) (5) Investing is a game of chance. Suppose there is a 34% chance that a risky stock investment will end up in a total loss of your investment. Because the rewards are so high, you decide to invest four independent risky stocks. Find the probability that at least one of your four investments becomes a total loss.

$P(\text{total loss}) = 0.34$

$P(\text{NOT total loss}) = 0.66$

$P(\text{at least 1 total loss})$

$= 1 - P(\text{0 total loss})$

$= 1 - (0.66)^4$

$= 0.81025$

OR Use Binomal

$n=4, p=.34, q=0.66$

X: # of total loss investment.

$P(X \geq 1)$

$= 1 - P(X=0)$

$= 1 - \text{binompdf}(4, .34, 0)$

$= 0.81025$

Find the indicated probability. Give your answer as a simplified fraction.

7) (12) The manager of a used car lot took inventory of the automobiles on his lot and constructed the following table based on the age of his car and its make (foreign or domestic).

Age of Car (in years)

Make	0 – 2	3 – 5	6 – 10	over 10	Total
Foreign	45	29	12	14	100
Domestic	36	28	15	21	100
Total	81	57	27	35	200

a. A car was randomly selected from the lot. Given that the car selected was a foreign car, what is the probability that it was older than 2 years?

$P(\text{older than 2 yrs} \mid \text{foreign Car}) = \frac{29+12+14}{100} = \frac{55}{100} = 0.55$

b. A car was randomly selected from the lot, find the probability that it was a foreign car.

$P(\text{foreign Car}) = \frac{100}{200} = \frac{1}{2}$

c. A car was randomly selected from the lot, find the probability that it was a foreign car or it was older than 10 years.

$P(\text{Foreign or older than 10})$

$= \frac{100}{200} + \frac{35}{200} - \frac{14}{200}$

$= \frac{121}{200}$ or $= 0.605$

d. Three cars were randomly selected from the lot, find the probability that they are all domestic cars.

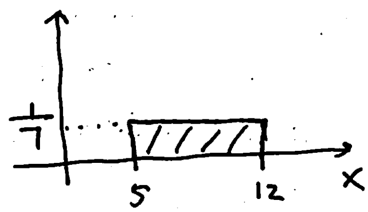
$P(\text{ALL 3 are domestic cars})$

$= \frac{100}{200} \cdot \frac{99}{199} \cdot \frac{98}{198}$

$= 0.123$

8) (6) Assume that the weight loss for the first month of a diet program varies between 5 pounds and 12 pounds, and is spread evenly over the range of possibilities, so that there is a **uniform distribution.**

a. Find the probability of a random person loss more than 8.5 pounds.



X: # of pounds.

$P(x)\cdot 7 = 1$

$P(x) = \frac{1}{7}$

P(more than 8.5 pounds)

$= P(X > 8.5)$

$= \frac{1}{7}(12 - 8.5)$

$= 0.5$

b. Find the probability that a random person loss between 6.5 and 10 pounds.

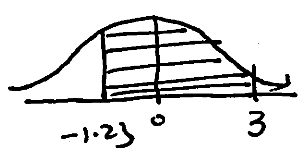
$P(6.5 < X < 10)$

$= \frac{1}{7}\cdot(3.5)$

$= 0.5$

(8) Draw the density curve and use a table of areas to find the specified area under the standard normal curve.

9) a. Find P(–1.23 < z < 3)

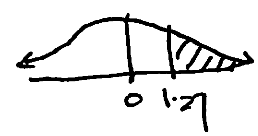


$P(-1.23 < z < 3)$

$= 0.9987 - 0.1093$

$= 0.8894$

b. Find P(z > 1.27)

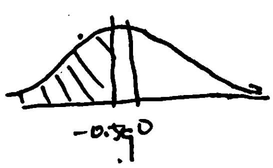


$P(z > 1.27)$

$= 1 - 0.8980$

$= 0.1020$

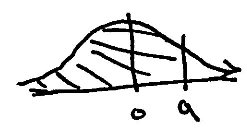
c. Find P(z < – 0.59)



$P(z < -0.59)$

$= 0.2776$

d. Given P(z < a) = 0.80, find a.



$a = 0.84$.

10) (5) Suppose you pay $2.00 to roll a fair die with the understanding that you will get back $5.00 for rolling a 1 or a 6, nothing otherwise. What is your expected value?

X: net profit.

	X	P(x)
win	3	$\frac{2}{6} = \frac{1}{3}$
lose	2	$\frac{4}{6} = \frac{2}{3}$

$E(x) = \sum X\cdot P(x)$

$= 3(\frac{1}{3}) - 2(\frac{2}{3})$

$= -0.33$

So, you expect to lose 33¢ for every $2.

11) (3, 3, 4) The cholesterol levels (in milligrams per deciliter) of 30 adults are listed below.

144	150	165	165	170	171	172	180	184	185
189	189	190	192	195	198	198	200	200	202
205	210	211	215	220	220	225	238	255	285

a. List the five-number summary.

Min = 144 $\quad$ $Q_1 = 180$

Max = 285 $\quad$ $Q_2 = 196.5$

$Q_3 = 211$

b. Find outliers, if any. Show your work. $Q_3 - Q_1 = IQR. \Rightarrow IQR = 211 - 180 = 31$

upper fence $= Q_3 + 1.5(IQR)$
$= 211 + 1.5(31)$
$= 257.5$

lower fence $= Q_1 - 1.5(IQR)$
$= 180 - 1.5(31)$
$= 133.5$

Hence, 285 is an outlier

c. Draw a boxplot that represents the data.

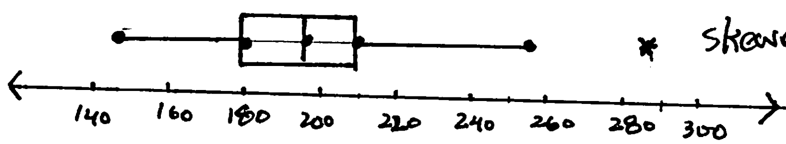
min = 144 $\quad$ outlier : 285

$Q_1 = 180$

$Q_2 = 196.5$

$Q_3 = 211$

max = 255



* skewed to the right.

12) (5) You toss a fair coin 5 times. What is the probability of at least one head?

$n = 5$ $\quad$ X: # of heads $\quad$ $P(X \geq 1) = 1 - P(X = 0)$

$p = 0.5$ $\quad$ $= 1 - (\frac{1}{2})^5$

$q = 0.5$ $\quad$ $= \frac{31}{32}$

13) (5) The probability that a football game will go into overtime is 18%. What is the probability that two of three football games will go to into overtime?

$n = 3$ $\quad$ X: # of overtime games $\quad$ $P(x = 2)$

$p = 0.18$ $\quad$ $=$ binompdf $(3, 0.18, 2)$

$q = 0.82$ $\quad$ $= 0.0797$

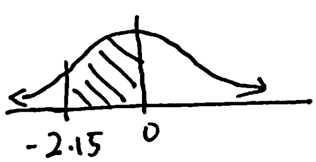
14) (4) **Draw the density curve, then find the area.**

a. Find the area under the standard normal curve to the right of z = 1.

$P(Z > 1)$

$= 0.1587$

b. Find the area under the standard normal curve bewteen z = – 2.15 and z = 0.



$P(-2.15 < Z < 0)$

$= 0.5 - 0.0158$

$= 0.4842$