Math54 Test #3 Name Seungmi Song **95**

Show all work neatly and systematically for full credit. Total points:100.
For confidence interval, you must clearly state the critical values, the margiin of error, the confidence interval, and a conclusion sentence.

1) (8) Leakage from underground fuel tanks has been a source of water pollution. In a random sample of 107 gasoline stations, 18 were found to have at least one leaking underground tank. Construct a 95% confidence interval for the proportion of gasoline stations with at least one leaking underground tank. Write a conclusion sentence.

$n = 107$
$x = 18$
$\hat{p} = \frac{18}{107} = 0.168$
$\hat{q} = 0.832$

① 

$Z_{\frac{\alpha}{2}} = 1.96$

② $E = 1.96 \times \sqrt{\frac{(0.168)(0.832)}{107}} = 0.0708$ ✓

③ $\hat{p} - E < p < \hat{p} + E$

$0.168 - 0.0708 < p < 0.168 + 0.0708$

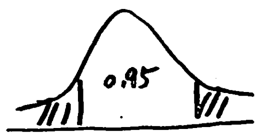
$0.0972 < p < 0.2388$ ✓

We are 95% confident that the interval from 0.0972 to 0.2388 contains the proportion of gasoline stations with at least one leaking underground tank.

2) (5) A research wantsto estimate the proportion of households that have broadbank Internet access. What size sample should be obtained if she wishes the estimate to be within 2% and with 95% confidence if she uses an estimate of 0.635 obtained from the National Telecommunications and Information Administration?

$E = 0.02$
$\hat{p} = 0.635$
$\hat{q} = 0.365$

$n = \frac{(Z_{\frac{\alpha}{2}})^2 \hat{p}\hat{q}}{E^2} = \frac{(1.96)^2(0.635)(0.365)}{(0.02)^2} \approx \boxed{2226}$ ✓



$Z_{\frac{\alpha}{2}} = 1.96$

3) (5) Furnace repair bills are normally distributed with a mean of \$272 and a standard deviation of \$25. If 100 of these repair bills are randomly selected, find the probability that they have a mean cost less than \$274.

$\mu = 272$
$\sigma = 25$

$P(x < 274) = P(z < 0.08) = \boxed{0.5319}$

$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{274 - 272}{25} = \cancel{0.08}$

-3

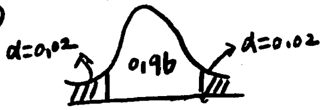
Correction: $P(\bar{x} < 274) = P\left(z < \frac{274 - 272}{\left(\frac{25}{\sqrt{100}}\right)}\right)$
$= P(z < 0.8)$
$= 0.7881$

-3

$E = 3$
$\sigma = 10$

4) (5) The principal at Riverside High School would like to estimate the mean length of time each day that it takes all the buses to arrive and unload the students. How large a sample is needed if the principal would like to assert with 96% confidence that the sample mean is off by, at most, 3 minutes? Assume that σ = 10 minutes based on previous studies.

①



$z_{\frac{\alpha}{2}} = 2.05$

$\alpha = 0.02 \simeq 0.0202$

$z_{\frac{\alpha}{2}} = 2.05$

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E}\right)^2 = \left(\frac{2.05 \times 10}{3}\right)^2 \simeq \boxed{47}$$

✓

5) (8) In a sample of 87 young adult, the average time per day spent in bed asleep was 7.06 hours and the standard deviation was 1.11 hours. Construct a 99% confidence interval for the mean time spent in bed asleep. Write a conclusion sentence.

$(n > 30)$
$n = 87$
$\bar{x} = 7.06$
$s = 1.11$

①

가까운 작은 (round off)

$1 - 0.99 = 0.01$, $df = 86$ $(0.01, 90)$

$t_{\frac{\alpha}{2}} = 2.632$ ✓

② $E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 2.632 \times \frac{1.11}{\sqrt{87}} = 0.313$ ✓

③ $\bar{x} - E < \mu < \bar{x} + E$

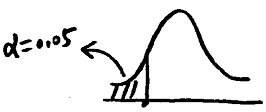
$7.06 - 0.313 < \mu < 7.06 + 0.313$

$6.747 < \mu < 7.373$ ✓

We are 99% confident that the interval from 6.747 hours to 7.373 hours contains the mean time spent in bed sleep.

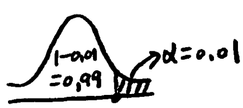
(6) **Find the critical value(s). Assume normal distribution.**

6) a. Determine the critical value. It is a left-tailed test of a population proportion at the α = 0.05 level of significance.



$\boxed{Z_{crit} = -1.645}$ ✓

b. Determine the critical value. Test the claim about the population proportion p > 0.28, and α = 0.01.
↳ right tailed



$\alpha = 0.99 \simeq 0.9901$ $Z = 2.33$

$\boxed{Z_{crit} = 2.33}$ ✓

c. Determine the critical values for a two-tailed test of a population proportion at the α = 0.05 level of significance.

$\alpha = 0.025 = 0.0250$

$\boxed{Z_{crit} = \pm 1.96}$ ✓

Identify the null hypothesis, alternative hypothesis, test statistic, P-value or critical value, make conclusion about the null hypothesiss and write final conclusion that addresses the original claim.

7) (8) A poll of 1068 adult Americans reveals that 48% of the voters surveyed prefer the Democratic candidate for the presidency. At the 0.05 level of significance, test the claim that at least half of all voters prefer the Democrat.

① Claim: $p \geq 0.50$ equal sign

$H_0: p = 0.50$

$H_1: p < 0.50$ left tailed

② $n = 1068$

$\hat{p} = 0.48$

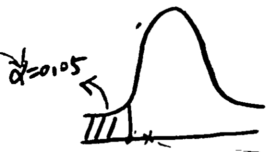
$p = 0.50$

$q = 0.50$

③ T.S

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.48 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{1068}}} = -1.307$$

④ $\alpha = 0.05$



$z_{crit} = -1.645$ not include

⑤ fail to reject H_0

There is not sufficient evidence at $\alpha = 0.05$ to warrant rejection of the claim that at least half of all voters prefer the Democrat.

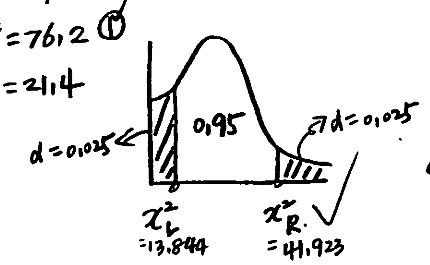
8) (8) A sociologist develops a test to measure attitudes about public transportation, and 27 randomly selected subjects are given the test. Their mean score is 76.2 and their standard deviation is 21.4. Construct the 95% confidence interval for the standard deviation, σ, of the scores of all subjects. Assume the population is normal.

chi-square

$n = 27$

$\bar{x} = 76.2$ ①

$s = 21.4$



$df = 26$

$\chi^2_R = 41.923$

$(0.025, 26)$

$\chi^2_L = 13.844$

$1 - 0.025 = 0.975$

$(0.975, 26)$

② $$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{26 \times (21.4)^2}{41.923}} < \sigma < \sqrt{\frac{26 \times (21.4)^2}{13.844}}$$

$$16.85 < \sigma < 29.33$$

③ We are 95% confident that the interval from 16.85 to 29.33 contains the population σ.

9) (3, 5 points)

The reading speed of second grade students is approximately normal with a mean of 90 words per minute and a standard deviation of 10 words per minutes.

a. What is the probability a randomly selected student will read more than 95 words per minute?

$\mu = 90$

$\sigma = 10$

$P(x > 95) = P(z > 0.5) = 1 - 0.6915 = \boxed{0.3085}$

$z = \frac{x-\mu}{\sigma} = \frac{95-90}{10} = 0.5$

b. What is the probability that a random sample of 24 second grade students results in a mean reading rate of more than 95 words per minute?

$\mu = \mu_{\bar{x}} = 90$

$\sigma_{\bar{x}} = \frac{10}{\sqrt{24}} = 2.04$

$P(x > 95) = P(z > 2.45) = 1 - 0.9929 = \boxed{0.0071}$

$z = \frac{95-90}{2.04} = 2.45$

mean, σ, p 잘봐야지

(6) Express the null hypothesis and the alternative hypothesis in symbolic form.

10) a. Carter Motor Company claims that its new sedan, the Libra, will average better than 26 miles per gallon in the city.

$\mu > 26$

$H_0 : \mu = 26$

$H_1 : \mu > 26$

b. A psychologist claims that more than 4.1 percent of the population suffers from professional problems due to extreme shyness.

$p > 4.1\%$

$H_0 : p = 4.1\%$

$H_1 : p > 4.1\%$

c. A researcher claims that the amounts of acetaminophen in a certain brand of cold tablets ~~have~~ have a standard deviation different from 3.3 mg claimed by the manufacturer.

$p = \sigma - 3.3$

$H_0 : p = \sigma - 3.3$

$H_1 : p \neq \sigma - 3.3$

σ

Correction :

claim : $\sigma \neq 3.3$

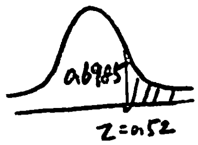
$H_0 : \sigma = 3.3$

$H_1 : \sigma \neq 3.3$

-2

(6) Use the given information to find the P-value.

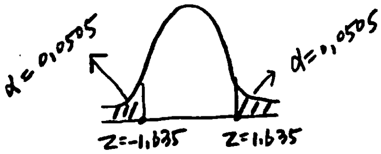
11) a. Assume normal distribution. The test statistic in a right-tailed test is z = 0.52. Find p-value.



$p\text{-value} = P(z > 0.52) = 1 - 0.6985 = \boxed{0.3015}$ ✓

✓ b. Assume normal distribution. It is a two-tailed test. with test statistic z = -1.635. Find p-value

-1.64



$p\text{-value} = 2 \cdot P(z < -1.635) = 2 \times 0.0505 = \boxed{0.101}$ ✓

12) (3, 5) Assume that adult females have pulse rates that are normally distributed with a mean of 74 bpm and a standard deviation of 12.5 bpm.

a. If one adult female is randomly selected, find the probability that her pulse rate is less than 75 bpm.

$\mu = 74$

$\sigma = 12.5$

$P(x < 75) = P(z < 0.08) = \boxed{0.5319}$

$z = \dfrac{x-\mu}{\sigma} = \dfrac{75-74}{12.5} = 0.08$ ✓

✓

b. If 16 adult females are randomly selected, find the probability that they have pulse rates with a mean less then 75 bpm.

$\mu = \mu_{\bar{x}} = 74$

$\sigma_{\bar{x}} = \dfrac{12.5}{\sqrt{16}} = 3.125$

$P(x < 75) = P(z < 0.32) = \boxed{0.6255}$

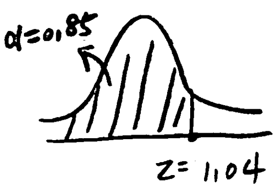
$z = \dfrac{x-\mu}{\sigma} = \dfrac{75-74}{3.125} = 0.32$ ✓

13) (5) Suppose that replacement times for washing machines are normally distributed with a mean of 9.5 years and a standard deviation of 1.8 years. Find the replacement time that separates the top 15% from the bottom 85%.

x

$\mu = 9.5$

$\sigma = 1.8$



$\alpha = 0.85 \approx 0.8508$

$z = \dfrac{x-\mu}{\sigma}$ ✓

$1.04 = \dfrac{x-9.5}{1.8}$ ✓

$x = 11.372$

$\boxed{11.372 \text{ years}}$ The replacement times for washing machines is 11.372 years.

14) (6) The football coach randomly selected ten players and timed how long each player took to perform a certain drill. The times (in minutes) were:

7.2 10.5 9.9 8.2 11.0

7.3 6.7 11.0 10.8 12.4

Determine a 95% confidence interval for the mean time for all players. Assume that the population has a normal distribution. Write a conclusion sentence.

$n = 10$

$\bar{x} = 9.5$

$s = 1.198$

① 

$1 - 0.95 = 0.05$, $df = 9$

$t_{\frac{\alpha}{2}} = 2.262$

② $E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 2.262 \times \frac{1.98}{\sqrt{10}} = 1.42$

③ $\bar{x} - E < \mu < \bar{x} + E$

$9.5 - 1.42 < \mu < 9.5 + 1.42$

$8.08 < \mu < 10.92$

We are 95% confident that the interval from 8.08 min to 10.92 min contains the mean time for all players.

Identify the null hypothesis, alternative hypothesis, test statistic, P-value or critical value, make conclusion about the null hypothesis, and write final conclusion that addresses the original claim.

15) (8) In a sample of 167 children selected randomly from one town, it is found that 37 of them suffer from asthma. At the 0.05 significance level, test the claim that the proportion of all children in the town who suffer from asthma is more than 15%.

① claim: $p > 0.15$

$H_0: p = 0.15$

$H_1: p > 0.15$ right tailed

② $n = 167$

$x = 37$

$\hat{p} = \frac{37}{167} = 0.222$

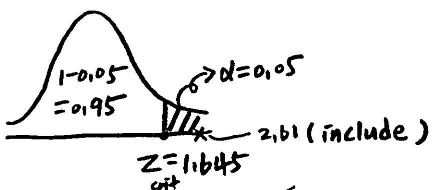
$p = 0.15$

$q = 0.85$

③ T.S

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.222 - 0.15}{\sqrt{\frac{(0.15)(0.85)}{167}}} = 2.61$$

④ $\alpha = 0.05$



⑤ reject H_0

There is sufficient evidence at $\alpha = 0.05$ to support the claim that the proportion of all children in the town who suffer from asthma is more than 15%