Math 227 **Test #2 (Ch 4 and 5)** **Name** Solution

Show all work neatly and systematically for full credit. Total points: 100

Find the indicated probability by using the special addition rule.

1) (3) A card is drawn from a well-shuffled deck of 52 cards. What is the probability of drawing a king or a red card?

$$P(\text{King or red}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52}$$
$$= \frac{28}{52}$$
$$= 0.538 \text{ or } \frac{7}{13}$$

Provide an appropriate response.

2) (8) In a recent survey, 60% of the community favored building a health center in their neighborhood.

a. If 14 citizens are chosen, find the probability that exactly 5 of them favor the building of the health center.

Binomial
$n=14$
$p=0.6$
$q=0.4$

X: # of people favored building a health Center.

$$P(X=5) = \text{binom pdf}(14, 0.6, 5)$$
$$= 0.0408$$

b. If 14 citizen are chosen, find the probability that at least one of them favor the building of the health center.

$$P(X \geq 1) = 1 - P(X=0)$$
$$= 1 - \text{binompdf}(14, 0.6, 0)$$
$$= 0.999997$$

3) (3) License plates are made using 2 letters followed by 2 digits.

a. How many plates can be made if repetition of letters and digits is allowed?

Number of license plate $= 26 \cdot 26 \cdot 10 \cdot 10$
$= 67{,}600$

b. How many plates can be made if repetition of letters and digits is not allowed?

Number of license plate $= 26 \cdot 25 \cdot 10 \cdot 9$
$= 58{,}500$

Find the mean of the random variable.

4) (9) The random variable X is the number of houses sold by a realtor in a single month at the Sendsom's Real Estate office. Its probability distribution is given in the table.

x	0	1	2	3	4	5	6	7
P(X = x)	0.24	0.11	0.12	0.16	0.21	0.14	0.01	0.01

a. Find the mean number of houses sold by a realtor in a single month.

$\mu = \sum x \cdot P(x) = 0(.24) + 1(.11) + 2(.12) + 3(.16) + 4(.21) + 5(.14) + 6(.01) + 7(.01)$

$= 2.5$

b. Find the standard deviation of the number of houses sold by a realtor in a single month.

$\sigma^2 = \sum x^2 \cdot P(x) - \mu^2$

$= 9.74 - 2.5^2$

$= 3.49$

$\sigma = \sqrt{3.49}$

$= 1.8682$

c. Is 6 a significantly hight number of houses sold by a realtor in a single month? Explain.

$2.5 + 2(1.8682)$

$= 6.2364$

since $6 < 6.2364$, 6 is not a significantly high number of houses sold.

5) (6) According to a college survey, 22% of all students work full time.

a. Find the mean and standard deviation for the number of students who work full time in samples of size 16.

Binomial $n = 16$

$p = .22$

$q = 0.78$

$\mu = np$

$= 16(0.22)$

$= 3.52$

$\sigma = \sqrt{npq}$

$= \sqrt{16(0.22)(0.78)}$

$= 1.6570$

b. Is the result of 14 students who work full time in samples of size 16 a result that is significantly high? Explain.

$\mu + 2\sigma = 3.52 + 2(1.6570)$

$= 6.834$

Yes, Since $14 > 6.834$, it would be significantly high.

Use the Poisson Distribution to find the indicated probability. Round to 3 significant digits.

6) (6) A computer salesman averages 1.7 sales per week. Use the Poisson distribution to

a. find the probability that in a randomly selected week the number of computers sold is 1.

$\mu = 1.7$ sales per week.

X: # of Computer sold.

$P(X = 1) = \text{poissonpdf}(1.7, 1)$

$= 0.3106$

b. find the probability that in a randomly selected week the number of computers sold is at least 2.

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - \text{poissoncdf}(1.7, 1)$

$= 0.507$

Find the indicated probability. Round to three significant digits.

7) (9) A multiple choice test has 30 questions, and each has four possible answers, of which one is correct.

a. If a student guesses on every question, find the probability of getting exactly 18 correct.

Binomial $n=30$ $\quad$ X: Number of Correct answers.

$p=\frac{1}{4}=0.25$ $\quad$ $P(X=18)=\text{binompdf}(30,0.25,18)$

$q=\frac{3}{4}=0.75$ $\quad$ $=3.987\times10^{-5}=0.00003987$

b. If a student guesses on every question, find the probability of getting at least 10 correct.

$P(X\geq10)=1-P(X\leq9)$

$=1-\text{binomcdf}(30,0.25,9)$

$=0.197$

c. If a student guesses on evry question, find the probability of getting at most 12 correct.

$P(X\leq12)=\text{binomcdf}(30,0.25,12)$

$=0.978$

Solve the problem.

8) (6) There are 12 members on a board of directors.

a. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible?

${}_{12}P_3=1320$

b. If they must form a subcommittee of 3 members, how many different ways are possible.

${}_{12}C_3=220$

Find the expected value of the random variable. Round to the nearest cent unless stated otherwise.

9) (5) Suppose you buy 1 ticket for $1 out of a lottery of 1,000 tickets where the prize for the one winning ticket is to be $500. What is your expected value?

	X net profit	P(x)
win	\$499	$\frac{1}{1000}$
lose	−\$1	$\frac{999}{1000}$

Expected value:

$E(X)=499\left(\frac{1}{1000}\right)-1\left(\frac{999}{1000}\right)$

$=-0.5$.

For every \$1 ticket, expected to lose ~~\$~~ 50¢.

10) (15) The manager of a used car lot took inventory of the automobiles on his lot and constructed the following table based on the age of his car and its make (foreign or domestic).

	Age of Car (in years)				
Make	0 – 2	3 – 5	6 – 10	over 10	Total
Foreign	37	28	13	22	100
Domestic	35	21	14	30	100
Total	72	49	27	52	200

a. A car was randomly selected from the lot. Given that the car selected was a domestic car, what is the probability that it was older than 2 years?

$$P(\text{older than } 2 \mid \text{domestic}) = \frac{21+14+30}{100} = \frac{65}{100} = 0.65$$

b. A car was randomly selected from the lot, what is the probability that it was older than 2 years?

$$P(\text{older than } 2\text{yrs}) = \frac{128}{200} = 0.64$$

c. A car was randomly selected from the lot, what is the probability that it was older than 10 years or was a foreign car?

$P(\text{older than } 10 \text{ yr } \underline{\text{OR}} \text{ foreign car})$

$$= \frac{52}{200} + \frac{100}{200} - \frac{22}{200}$$

$$= \frac{130}{200} = 0.65$$

d. A car was randomly selected from the lot, what is the probability that it was older than 10 years and was a domestic car?

$P(\text{older than } 10 \text{ yrs } \underline{\text{and}} \text{ domestic car})$

$$= \frac{30}{200} = 0.15$$

e. If three cars were randomly selected from the lot, what is the probability that all of them were foreign cars?

$$P(\text{ALL 3 are foreign cars}) = \frac{100}{200} \cdot \frac{99}{199} \cdot \frac{98}{198} = 0.123$$

OR $P(\text{ALL 3 are foreign cars})$

$$= \left(\frac{100}{200}\right)^3 = 0.125$$

Find the indicated probability.

11) (5) In a batch of 8000 clock radios 5% are defective. A sample of 15 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be rejected if at least one of those tested is defective. What is the probability that the entire batch will be rejected?

$n = 15$

$P(\text{defective}) = 0.05$

$P(\text{good}) = 0.95$

$P(\text{at least 1 defective})$

$= 1 - P(\text{ALL good})$

$= 1 - (0.95)^{15}$

$= 0.537$

Solve the problem.

12) (6) A pool of possible jurors consists of 10 men and 16 women.

a. How many different juries consisting of 5 men and 7 women are possible?

10 men 16 women
26 Total

$${}_{10}C_5 \cdot {}_{16}C_7 = 2,882,880$$

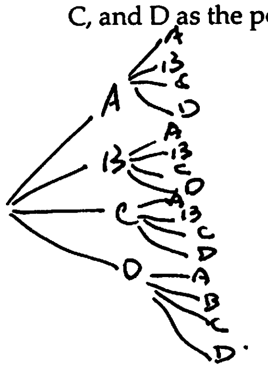
b. Find the probability of selecting juries consisting of all women.

P(ALL women)

$$= \frac{\text{\# of ways to select ALL women}}{\text{\# of ways to select 12 people}} = \frac{{}_{16}C_{12}}{{}_{26}C_{12}} = \frac{1820}{9,657,700} = 0.000188$$

Provide an appropriate response.

13) (4) Identify the sample space of the probability experiment: answering two multiple choice questions with A, B, C, and D as the possible answers



{AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD, DA, DB, DC, DD}

$4 \cdot 4 = 16$ possible outcomes.

14) (12) **Birthday Problems:**

a. Find the probability that a randomly slected student has birthday on Jan. 14.

$$P(\text{birthday on } 1/14) = \frac{1}{365}$$

b. Find the probability that of 5 randomly selected students, no two share the same birthday.

$$P(\text{No 2 share same birthday}) = P(\text{ALL different birthday})$$

$$= \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365}$$

$$= 0.973$$

c. Find the probability that of 5 randomly selected students, they all have same birthday.

$$P(\text{ALL have same birthday}) = \frac{365}{365} \cdot \frac{1}{365} \cdot \frac{1}{365} \cdot \frac{1}{365} \cdot \frac{1}{365}$$

$$= 5.63 \times 10^{-11}$$

d. Find the probability that of 5 randomly selected students, at least two share the same birthday.

$$P(\text{At least 2 share same birthday}) = 1 - P(\text{ALL different birthday})$$

$$= 1 - 0.973$$

$$= 0.027$$