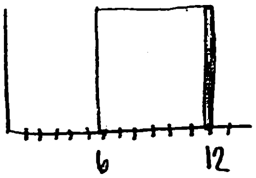
Math 227 Test #3 Name Varai Godinez

98 + 5

Show all work neatly for full credit.
For confidence interval, state the critical values and the interval. For hypotheses test, show all steps. Total points: 104

1) (5) Assume that the weight loss for the first month of a diet program varies between 6 pounds and 12 pounds, and is spread evenly over the range of possibilities, so that there is a uniform distribution. Find the probability of a randomly selected person loss more than 11 pounds for the first month of the diet program.



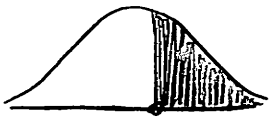
range × probability

$P(X>11) = 1 \times \frac{1}{6}$

$\boxed{= .167}$

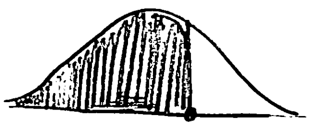
(12) If z is a standard normal variable, draw the density curve and find the probability.

2) a. $P(z > 0.51)$



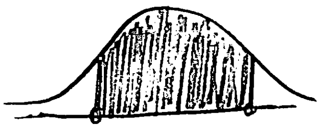
$= .3050$

b. $P(z < 1.21)$



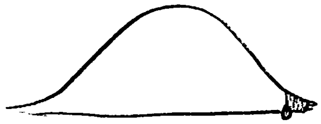
$= .8869$

c. $P(-0.72 < z < 2.07)$



$.9808 - .2358 = .7450$

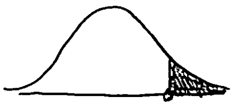
d. $P(z > 5.61)$



$= .0001$

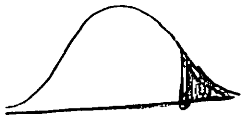
(6) Find the indicated value. Draw a density curve for each.

3) a. $z_{0.005}$



$= 2.575$

b. $z_{0.10}$



$= 1.28$

(5) Find the indicated probability.

4) The incomes of trainees at a local mill are normally distributed with a mean of \$1100 and a standard deviation of \$150. What percentage of trainees earn less than \$900 a month?

$\mu = 1100$
$\sigma = 150$

$P(X < 900)$

$P\left(z < \frac{900-1100}{150}\right)$

$P(z < -1.33) \Rightarrow$

$= .0918$

9.18% of trainees earn less than \$900 a month

(6) Provide an appropriate response.

5) Flood insurance policies sold in the last three days by a new agent were 3, 5, and 7. Assume that samples of size 2 are randomly selected with replacement from this population of three values. List the different possible samples, and find the mean of each of them. Construct the probability distribution of sample means, then find the mean and the standard deviation of the probability distribution.

3,3	3
3,5	4
3,7	5
5,3	4
5,5	5
5,7	6
7,3	5
7,5	6
7,7	7

$\bar{X}$	$P(\bar{X})$	$\bar{X}\cdot P(\bar{X})$	$\bar{X}^2\cdot P(\bar{X})$
3	1/9	1/3	1
4	2/9	8/9	32/9
5	3/9	5/3	25/3
6	2/9	4/3	8
7	1/9	7/9	49/9
		$\Sigma = 5$	$\Sigma = 26.33$

$\mu = 5$

$\sigma = \sqrt{26.33 - (5)^2}$

$= 1.15$

(5) Solve the problem.

6) The weights of the fish in a certain lake are normally distributed with a mean of 19 lb and a standard deviation of 6. If 9 fish are randomly selected, what is the probability that the mean weight will be between 16.6 and 22.6 lb?

CLT

$\mu = 19$

$\sigma = 6$

$X = 9$

$\mu_{\bar{x}} = 19$

$\sigma_{\bar{x}} = \frac{6}{\sqrt{9}} = 2$

$P(16.6 < \bar{x} < 22.6)$

$P\left(\frac{16.6-19}{2} < z < \frac{22.6-19}{2}\right)$

$P(-1.2 < z < 1.8)$

$.9641 - .1151 = \boxed{.849}$

(5) Find the indicated probability.

7) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. If an applicant is randomly selected, find the probability of a rating that is between 170 and 220.

$\mu = 200$

$\sigma = 50$

$P(170 < X < 220)$

$P\left(\frac{170-200}{50} < X < \frac{220-200}{50}\right)$

$P(-0.6 < X < 0.4)$

$.6554 - .2743 = \boxed{.3811}$

8) (5) A final exam in Math 160 has a mean of 63 with standard deviation 7.8. If 49 students are randomly selected, find the probability that the mean of their test scores is greater than 61.

$\mu = 63$

$\sigma = 7.8$

$n = 49 \leq 30$ ✓

CLT $\mu_{\bar{x}} = 63$

$\sigma_{\bar{x}} = \frac{7.8}{\sqrt{49}} = 1.11$

$P(\bar{X} > 61)$

$1 - P(\bar{X} \leq 61)$

$1 - P\left(z \leq \frac{61-63}{1.11}\right)$ ✓

$1 - P(z \leq -1.80)$

$1 - .0359 = \boxed{.9641}$ ✓

9) (5) In a certain population, body weights are normally distributed with a mean of 152 pounds and a standard deviation of 26 pounds. How many people must be surveyed if we want to estimate the percentage who weigh more than 180 pounds? Assume that we want 95% confidence that the error is no more than 4 percentage points.

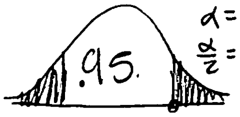
$E \leq .04$

$\mu = 152$

$\sigma = 26$

$X > 180$

$CL: 95\%$



$z_{\frac{\alpha}{2}}$

$\alpha = .05$

$\frac{\alpha}{2} = .025$

$= 1.96$

$n = \frac{(1.96)^2 \, 0.25}{(.04)^2} = 601$ ✓

10) (6) A certain question on a test is answered correctly by 22% of the respondents. Use normal distribution as an approximation to the binomial distribution to estimate the probability that among the next 150 responses there will be at most 40 correct answers.

$p = .22$

$q = .78$

$n = 150$

$X = 40$ ✓

$np = 150(.22) = 33 \geq 5$ ✓

$nq = 150(.78) = 117 \geq 5$ ✓ ✓

$\mu = 33$ ✓

$\sigma = \sqrt{(150)(.22)(.78)}$

$= 5.07$ ✓

$P(X \leq 40)$ ✓

$P_c(X \leq 40.5)$ ✓

$P\left(z \leq \frac{40.5 - 33}{5.07}\right)$ ✓

$P(z \leq 1.44) = \boxed{.9251}$ ✓ ✓

n $\qquad$ x

11) (6) Of 88 adults selected randomly from one town, 69 have health insurance. Find a 90% confidence interval for the true proportion of all adults in the town who have health insurance.

$n = 88 \geq 5$

$x = 69$

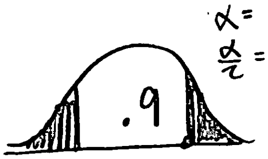
CL: 90%

$\hat{p} = \frac{69}{88} = .784$

$\hat{q} = .216$

$\alpha = .1$

$\frac{\alpha}{2} = .05$



1.645

$E = (1.645)\sqrt{\frac{(.784)(.216)}{88}} = .0722$

$.784 - .0722 < p < .784 + .0722$

$.7118 < p < .8562$

we are 90% confident that the interval from .7118 to .8562 contains the true proportions of adults with health insurance in the town

12) (3) Assume that t distribution applies. Find the critical value $t_{\alpha/2}$ that corresponds to a 90% confidence level.

$n = 20$

$n = 20$

$df = 19$

CL = 90%

$\boxed{t_{\frac{\alpha}{2}} = 1.729}$

13) (5) How many business students must be randomly selected to estimate the mean monthly earnings of business students at one college? We want 95% confidence that the sample mean is within \$128 of the population mean, and the population standard deviation is known to be \$536.

σ $\qquad$ E

$E = 128$

$\sigma = 536$

CL 95% = 1.96

$n = \left[\frac{(1.96)(536)}{128}\right]^2 = 68$

90%

14) (4) Find the critical value χ^2_R and χ^2_L corresponding to a sample size of 15 and a confidence level of 9 percent.

n

2 tails

$df = 14$

$\boxed{\chi^2_L = 6.571 \quad \chi^2_R = 23.685}$

15) (6) The principal randomly selected six students to take an aptitude test. Their scores were:

76.5 85.2 77.9 83.6 71.9 88.6

Determine a 90% confidence interval for the mean score for all students. Assume that the population has a normal distribution.

$n = 6$

$s = 6.23$

$CL = 90\%$

$df = 5$

$\bar{X} = 80.62$

$C.V. = 2.015$

$E = (2.015)\dfrac{6.23}{\sqrt{6}} = 5.12$

$80.62 - 5.12 < \mu < 80.62 + 5.12$

$75.5 < \mu < 85.74$

we are 90% confident that the interval from 75.5 to 85.74 contains the mean score for all students.

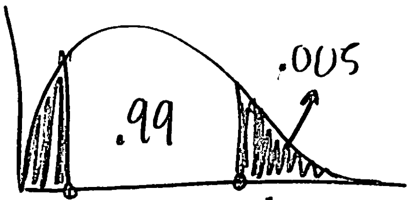
$s \quad n \quad \bar{X}$

16) (6) The mean replacement time for a random sample of 20 washing machines is 9.4 years and the standard deviation is 2.6 years. Construct a 99% confidence interval for the standard deviation, σ, of the replacement times of all washing machines of this type.

$n = 20$

$s = 2.6$

$df = 19$



$$\sqrt{\frac{(19)(2.6)^2}{38.582}} < \sigma < \sqrt{\frac{(19)(2.6)^2}{6.844}}$$

$1.825 < \sigma < 4.332$

we are 99% confident that the standard deviation of all washing machins of this type is between 1.825 and 4.332

(4) Express the null hypothesis and the alternative hypothesis in symbolic form. Use the correct symbol (μ, p, σ) for the indicated parameter.

17) (a) Carter Motor Company claims that its new sedan, the Libra, will average better than 26 miles per gallon in the city. Use μ, the true average mileage of the Libra.

claim = $\mu > 26$

$H_0: \mu = 26$

$H_1: \mu > 26$ ✓

(b). A psychologist claims that more than 4.1 percent of the population suffers from professional problems due to extreme shyness. Use p, the true percentage of the population that suffers from extreme shyness.

claim : $p > 4.1$

$H_0: p = 4.1\%$

-1 $H_1: p > 4.1\%$ ✓

(10) Identify the null hypothesis, alternative hypothesis, test statistic, critical value, conclusion about the null hypothesis, and final conclusion that addresses the original claim.

18) A supplier of digital memory cards claims that no more than 1% of the cards are defective. In a random sample of 600 memory cards, it is found that 3% are defective, but the supplier claims that this is only a sample fluctuation. At the 0.01 level of significance, test the supplier's claim that no more than 1% are defective.

n

-5

$n = 600$

$\hat{p} = .03$

$p = .01$

$q = .99$

$\alpha = .01$

claim : $p \le .01$ ✓

$\boxed{H_0: p = .01}$ reject

$H_1: p > .01$ ✓

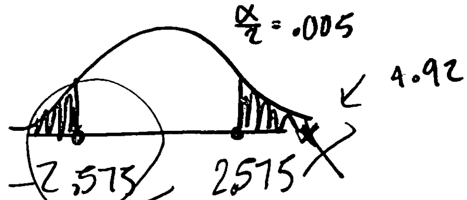
$np = 600(.01) = 6$ ✓

$nq = 600(.99) = 594$ ✓

$$z = \frac{.03 - .01}{\sqrt{\frac{(.01)(.99)}{600}}} = 4.92$$

$\alpha = .01$

$\frac{\alpha}{2} = .005$



reject H_0

conclusion:

There is enough evidence . . . .

I didn't have my own chart so I don't know how to write the conclusion for a rejection w/ equality

-6

#18) $n = 600$

$\hat{p} = 3\% = 0.03$

$\hat{q} = 0.97$

① claim: $p \le 1\% \Rightarrow p \le 0.01$

$H_0: p = 0.01$

$H_1: p > 0.01$

$\left.\begin{array}{l} np = 600(0.01) = 6 \ge 5 \\ nq = 600(0.99) = 594 \ge 5 \end{array}\right\}$ use normal dist.

② $Z = \dfrac{0.03 - 0.01}{\sqrt{\dfrac{(0.01)(0.99)}{600}}} = 4.92$

③ C.V. $\alpha = 0.01$

0.01

C.V. ~~$Z = 1.645$~~

$Z = 2.33$

p-value

4.92

p-value $= P(z > 4.92)$

$=$ normalcdf$(4.92, E99)$

$= 4.33 \times 10^{-7}$

④ Reject H_0

There is sufficient evidence at $\alpha = 0.01$ to warrant rejection of the claim that no more than 1% of the cards are defective.